

# Making Fuzzy Logic Work for Language

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Fuzzy Logic has been regarded as unsuitable for linguistic semantics since work by Kamp and Fine in the 1970s. In recent work, though, formal systems very similar to fuzzy logic have been explored by a number of researchers in the field (e.g. Lasersohn 1999; Smith 2009). In this talk, we argue for a semantics of natural language based on fuzzy logic, but enriched by an operation we call RESCALE. Rescaling of predicate is a way of making sure that the predicate assigns a broad range of truth values between 0 and 1 to the entities in its domain. We argue that the restriction of adjectives as in *tall for a jockey*, typicality with conjunctions, and local contradictions as in *tall and not tall* should be analyzed using rescaling. We argue that rescaling should apply recursively, and thereby is a part of sentence semantics, not pragmatics. With the addition of rescale, we indeed are beyond fuzzy logic, but the resulting system seems more suitable for natural language semantics than either classical or fuzzy logic. It also compares favorably to Bayesian and Quantum logical accounts.

**Adjective Restriction** We assume that an adjective like *tall* is a mapping from individuals to the interval  $[0,1]$  (i.e. of type  $\langle e, t \rangle$  in a fuzzy logic based semantics). Specifically, *tall* maps an entity of height 0mm to 0 and an entity of infinite height mapped to 1 (both 0 and 1 may be only attained in the limit if there are no entities of 0 or infinite height). In actual use, however, we assume that *tall* is usually restricted to a subset of all entities by covert domain restriction (cf. Stanley and Szabo 2000). In English, *for*-phrases express this restriction overtly (Kennedy 2007 and others). When combined with an argument as in (1), a restricted adjectives triggers a presupposition that the argument satisfy the restriction.

(1) Nadia is(n't) tall for a jockey. (presupposition: Nadja is a jockey)

A second effect of *for*-phrases is that the minimum height for *tall* can change – being tall for a jockey and being tall for basketball player entail different absolute heights. We propose that the apparent reset of the comparison class arise from an obligatory application of RESCALE. The most important aspect of RESCALE is that it maps a fuzzy property  $\phi$  isomorphically to another such that the image of  $\text{RESCALE}(\phi)$  is at least the open interval  $(0, 1)$ . We return to the detailed specification of RESCALE below.

(2) RESCALE is a functor of type  $\langle \langle \alpha, t \rangle, \langle \alpha, t \rangle \rangle$ . For any  $\phi$  of type  $\langle \alpha, t \rangle$ , the following three properties hold of  $\psi = \text{RESCALE}(\phi)$ :

- a. if  $\phi(x) > \phi(y)$ , then  $\psi(x) > \psi(y)$
- b.  $\inf(\{\psi(x) \mid x \in \text{domain}(\phi)\}) = 0$
- c.  $\sup(\{\psi(x) \mid x \in \text{domain}(\phi)\}) = 1$

Beyond these three axiomatic properties, we leave the definition of RESCALE flexible. Solt (2012) argue against simple linear interpolation for the definition of RESCALE.

**Conjunction Effects** Rescaling addresses one criticism by Kamp and Partee (1995) of fuzzy logic based semantics. They observe that fuzzy logic predicts that e.g. if  $x$  is an apple to degree  $\delta$  it cannot be a brown apple to a degree exceeding  $\delta$ . Intuitively though an typical brown apple would be atypical as an apple (see also Sassoon 2008). We propose that *typical* as in (3) involves rescaling of a property  $\phi$  restricted to individuals which satisfy  $\phi$  above some threshold  $\theta$  as in (4).

(3) This is a typical brown apple.

$$(4) \quad \llbracket \text{typical} \rrbracket(\phi) = \text{RESCALE}(\lambda x \in \{x \mid \phi(x) > \theta\} . \phi(x))$$

**Borderline Contradictions** Alxatib and Pelletier (2011); Ripley (2011); Sauerland (2011) demonstrated experimentally the acceptability of logical contradictions of a type that Ripley calls borderline contradictions. For ease of exposition we focus on acceptable borderline contradictions with individual predication such as (5) with an overt *for*-phrase:

(5) John, who is 5'10", is and isn't tall for an American male.

We assume that *and* and *not* have the fuzzy semantics in (6):

$$(6) \quad \begin{array}{l} \text{a.} \quad \llbracket \text{and} \rrbracket(\phi)(\psi) = \lambda x \in \text{domain}(\phi) \cap \text{domain}(\psi) . \min\{\phi(x), \psi(x)\} \\ \text{b.} \quad \llbracket \text{not} \rrbracket(\phi) = \lambda x \in \text{domain}(\phi) . 1 - \phi(x) \end{array}$$

In addition, we assume that RESCALE must apply after the coordination *and* is evaluated. Rescaling of the borderline contradiction

$$(7) \quad \text{RESCALE}(\phi \text{ and not } \phi)(\text{Joe}) = 1$$

**Recursive rescale** The simplest application conditions predict that RESCALE applies recursively in (8a) (shown in (8b)). Though the two embedded occurrences don't have an effect on global truth values, alternative application would be prohibitively complex.

(8) a. Joe is tall for a jockey and not tall for a jockey.  
b. Joe is RESCALE(RESCALE(tall for a jockey) and not RESCALE(tall for a jockey))

Examples such as (9) (following Kennedy 2007) further underscore the integration of RESCALE into sentential semantics.

(9) Everybody in my family RESCALE(is and isn't tall for somebody of their age)

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