Generalized Quantifiers on Dependent Types: A System for Anaphora

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In this paper we propose a system combining generalized quantifiers (Mostowski, 1957; Lindström, 1966; Barwise and Cooper, 1981) with dependent types (Martin-Löf, 1972; Ranta, 1994) for the interpretation of unbound anaphora.

Unbound anaphora. The phenomenon of unbound anaphora refers to instances where anaphoric pronouns occur outside the syntactic scopes of their quantifier antecedents. The main kinds of unbound anaphora are regular anaphora to quantifiers, quantificational subordination, and 'donkey anaphora', as exemplified by (1) to (3) respectively:

(1) Most kids entered. They looked happy.
(2) Every man loves a woman. They kiss them.
(3) Every farmer who owns a donkey beats it.

Unbound anaphoric pronouns have been dealt with in two main semantic paradigms: dynamic semantic theories (Groenendijk and Stokhof, 1991; Van den Berg, 1996; Nouwen, 2003) and the E-type/D-type tradition (Evans, 1977; Heim, 1990; Elbourne, 2005). In the dynamic semantic theories pronouns are taken to be (syntactically free, but semantically bound) variables, and context serves as a medium supplying values for the variables. In the E-type/D-type tradition pronouns are treated as quantifiers. Our system combines aspects of both families of theories. As in the E-type/D-type tradition we treat unbound anaphoric pronouns as quantifiers; as in the systems of dynamic semantics context is used as a medium supplying (possibly dependent) types as their potential quantificational domains.

The **main features** of our system are: (i) context and type dependency; and (ii) generalized quantifiers together with operations that lift quantifier phrases to chains of quantifiers, i.e. polyadic quantifiers (Bellert and Zawadowski, 1989; Benthem, 1989).

Context, types and dependent types. The variables of our system are always typed. We write $x : X$ to denote that the variable x is of type X and refer to this as a type specification of the variable x. Types are interpreted as sets. We write the interpretation of the type X as $\|X\|$. Types can depend on variables of other types. Thus, if we already have a type specification $x : X$, then we can also have type $Y(x)$ depending on the variable x and we can declare a variable y of type Y by stating $y : Y(x)$. The fact that Y depends on X is modeled as a projection $\pi : \|Y\| \to \|X\|$. So that if the variable x of type X is interpreted as an element $a \in \|X\|$, $\|Y\|(a)$ is interpreted as the fiber of π over a, i.e.: $\|Y\|(a) = \{b \in \|Y\| : \pi(b) = a\}$. Such type dependencies can be nested, i.e., we can have a sequence of type specifications of the (individual) variables: $x : X, y : Y(x), z : Z(x, y)$.

Context for us is a partially ordered sequence of type specifications of the (individual)

variables and it is interpreted as a parameter space, i.e. as a set of compatible n-tuples of elements of the sets corresponding to the types involved (compatible wrt all projections).
Quantifiers, chains of quantifiers. Our system defines quantifiers and predicates polymorphically. A generalized quantifier Q is an association to every set Z a subset of the power set of Z. The interpretation $\|P\|$ of an n-ary predicate P associates to a tuple of sets $\vec{Z} = \langle Z_1, \ldots, Z_n\rangle$ a subset of the cartesian product of the sets involved. Quantifier phrases, e.g. *some woman*, are interpreted as follows: $\|some_{w:woman}\| = \{X \subseteq \|woman\| : X \neq \emptyset\}$. The interpretation of quantifier phrases is further extended into the interpretation of chains of quantifiers. Chains of quantifiers are built from quantifier phrases using three chain-constructors: pack-formation rule $(?, \ldots, ?)$, sequential composition $?|?$, and parallel composition $\frac{?}{?}$. In our system the three chain-constructors and the corresponding semantical operations (known as cumulation, iteration and branching) are extended to (pre-) chains defined on dependent types. Finally, we say that a sentence $Ch_{\vec{y}:\vec{Y}}\ P(\vec{y})$ is true iff $\|P\|(\|\vec{Y}\|) \in \|Ch_{\vec{y}:\vec{Y}}\|$.
Dynamic extensions of contexts. To illustrate the process of the dynamic extensions of contexts, consider an example in (2). In our system language expressions (i.e. quantifiers, quantifier phrases, predicates, chains, and sentences) are all defined in context. The first sentence in (2) (on the most natural interpretation where *a woman* depends on *every man*) translates into a sentence in a context: $\Gamma \vdash \forall_{m:M}|\exists_{w:W} L(m,w)$. The way to understand the second sentence in (2) is that every man kisses the women he loves rather than those loved by someone else. Thus the first sentence in (2) must deliver some internal relation between the types corresponding to the two quantifier phrases. In our system the first sentence in (2) extends the context Γ by adding new variable specifications on newly formed types for every quantifier phrase in the chain Ch - for the purpose of the formation of such new types we introduce a new **type constructor** $\mathbb{T}$. That is, the first sentence in (2) (denoted as φ) extends the context by adding: $t_{\varphi,\forall_m} : \mathbb{T}_{\varphi,\forall_{m:M}}$; $t_{\varphi,\exists_w} : \mathbb{T}_{\varphi,\exists_{w:W}}(t_{\varphi,\forall_m})$.

The interpretation of types (that correspond to the quantifier phrases in the chain Ch) from the extended context Γ_φ are defined in a two-step procedure using the inductive clauses through which we define Ch but in the reverse direction.
Step 1. We define fibers of new types by inverse induction.
Basic step. For the whole chain $Ch = \forall_{m:M}|\exists_{w:W}$ we put: $\|\mathbb{T}_{\varphi,\forall_{m:M}|\exists_{w:W}}\| := \|L\|$.
Inductive step.

$$\|\mathbb{T}_{\varphi,\forall_{m:M}}\| = \{a \in \|M\| \,:\, \{b \in \|W\| \,:\, \langle a, b\rangle \in \|L\|\} \in \|\exists_{w:W}\|\}$$

and for $a \in \|M\|$

$$\|\mathbb{T}_{\varphi,\exists_{w:W}}\|(a) = \{b \in \|W\| \,:\, \langle a, b\rangle \in \|L\|\}$$

Step 2. We build dependent types from fibers.

$$\|\mathbb{T}_{\varphi,\exists_{w:W}}\| = \bigcup\{\{a\} \times \|\mathbb{T}_{\varphi,\exists_{w:W}}\|(a) : a \in \|\mathbb{T}_{\varphi,\forall_{m:M}}\|\}$$

Thus the first sentence in (2) extends the context by adding the type $\mathbb{T}_{\varphi,\forall_{m:M}}$, interpreted as $\|\mathbb{T}_{\varphi,\forall_{m:M}}\|$ (i.e. the set of men who love some women), and the dependent type $\mathbb{T}_{\varphi,\exists_{w:W}}(t_{\varphi,\forall_m})$, interpreted for $a \in \|\mathbb{T}_{\varphi,\forall_{m:M}}\|$ as $\|\mathbb{T}_{\varphi,\exists_{w:W}}\|(a)$ (i.e. the set of women loved by the man a). **Unbound anaphoric pronouns** are interpreted with reference to the context created by the foregoing text: they are treated as universal quantifiers and newly formed (possibly dependent) types incrementally added to the context serve as their potential quantificational domains. That is, unbound anaphoric pronouns $they_m$ and $them_w$ in the second sentence of (2) have the ability to pick up and quantify universally over the respective interpretations, yielding the correct truth conditions *Every man kisses every woman he loves.*

Empirically, our system allows a uniform treatment of both regular anaphora to quantifiers and the notoriously difficult cases such as quantificational subordination, 'donkey anaphora', and also cumulative and branching continuations.

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