# Few More and Many Fewer:
# Complex Quantifiers Based on *Many* and *Few*

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## 1. Introduction

This work examines the behavior of *many* and *few* in complex quantifiers, focusing on how they may be modified by degree modifiers such as *very*, *so*, *too* and comparative morphology, while also serving themselves as modifiers. I investigate the implications of these patterns for the semantics of *many*, *few* and the elements they combine with, and relate these cases to instances of modification in the adjectival domain.

## 2. The Data

I take as my starting point the patterns exemplified in (1)-(3):

(1) a. Many students attended the lecture
b. Few students attended the lecture

(2) a. Very many/very few students attended the lecture
b. Professor Jones was worried that too many/too few students would attend the lecture
c. The speaker was astonished that so many/so few students attended the lecture
d. More than 100/fewer than 100 students attended the lecture

(3) a. There were 100 seats in the lecture hall, but unfortunately many more than 100 students showed up for the lecture
b. The lecture hall has 500 seats, but few more than 100 students attended the lecture
c. The whole class of 100 was supposed to attend the lecture, but many fewer than 100 students actually came

*Many* and *few* may appear without modification, as in (1), or in combination with degree modifiers such as *very*, *so*, *too* and comparative morphology, as in (2) (where *more* is taken to be the spell-out of *many+er*). These are of course the very same modifiers that combine with gradable adjectives such as *tall* (*very tall*; *so tall*; *too tall*; *taller than 6 feet*), a parallel which has led many researchers to conclude that *many* and *few* are like gradable adjectives in requiring a semantic representation that references degrees on a scale associated with some dimension (Heim 2000, 2006; Hackl 2000; Kennedy 2001; Kennedy & McNally 2005; Rett 2006). In the case of *tall*, the relevant dimension is HEIGHT; in the case of *many/few*, it is CARDINALITY.

What has been less recognized (though see Schwarzschild 2006) is that *many* and *few* may themselves serve as modifiers in comparatives, giving us the expressions *many more*, *few more* and *many fewer* (though oddly no **few fewer*), as in (3), and in the naturally occurring examples in (4):

(4) a. Nearly 4 million Afghan children are enrolled in school, including more than 1 million girls, many more than at any point in Afghanistan's history
b. Few more than 400 Sumatran tigers survive in the wild
c. The latest attempt to count the number of transient vacation rentals on Maui finds many fewer than previously estimated

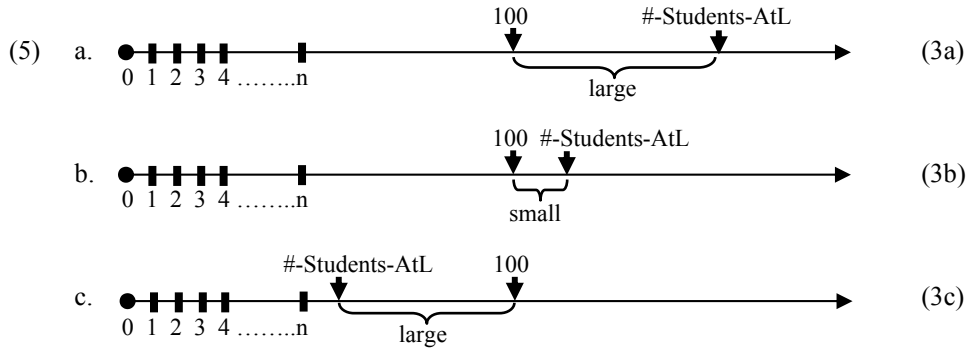
The existence of these sequences poses an immediate challenge to a classical Generalized Quantifier (GQ) analysis (Barwise & Cooper 1981) under which *many* and *few* and the complex quantifiers based on them are taken to be 'quantificational determiners' that denote relationships between two sets of individuals. In the examples in (3), *many*/*few* cannot be of semantic type ⟨et,⟨et,t⟩⟩ or its gradable counterpart ⟨d,⟨et,⟨et,t⟩⟩⟩ (Hackl 2000), because the logical form does not provide two appropriate sets to serve as arguments. Nor can comparatives such as *more than 100* and *fewer than 100* have the semantics of simple quantificational determiners, since this would preclude their modification by *many/few*.

It also bears mentioning that the facts in (3) cannot be easily captured with an analysis of *many* and *few* as gradable predicates of individuals (type ⟨d,et⟩), the type commonly ascribed to gradable adjectives (Heim 2000), and which some authors have suggested can be extended to *many* and *few* (Kennedy & McNally 2005; Hackl 2006): In (3a,b) we could perhaps view *many* and *few* as being predicated of the group of students in excess of the first 100 who attended the lecture; but in (3c), involving *many fewer*, there is no equivalent group of individuals to whom the property of 'many-ness' could be ascribed.

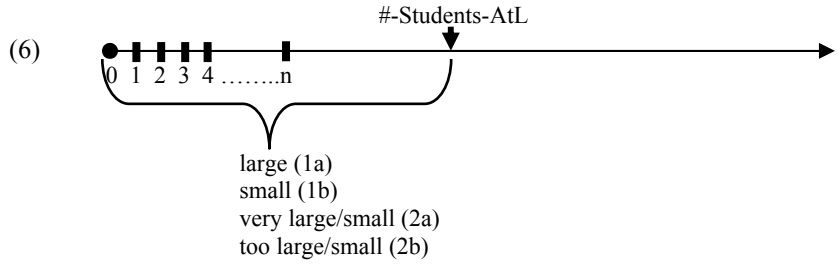
The conclusion is that neither classical GQ theory nor a direct extension of recent analyses of gradable adjectives can adequately account for the full distribution of *many* and *few*, as exemplified in (1)-(3). In the next section, I outline an alternate proposal that is able to capture the full range of facts, and in the following section I discuss its implications for the semantics of degree modifiers.

**3. The Proposal**

Intuitively, in the complex quantifiers in (3), *many* and *few* can be interpreted as expressing properties of intervals on the scale of natural numbers: In (3a), the interval from 100 to the number of students attending the lecture (henceforth **[100, #-Students-AtL]**) is characterized as large; in (3b) that interval is described as small; and in (3c), the interval [**#-Students-AtL, 100]** is described as large. This is shown schematically in (5):




While perhaps not as intuitively obvious, the occurrences of *many* and *few* in the simpler constructions in (1)-(2) can also be analyzed as expressing properties of intervals, in these cases the interval from 0 to the number of students who attended the lecture (**[0, #-Students-AtL]**). Thus in (1) this interval is characterized as large (for *many*) or small (for *few*) by some contextually relevant standard; in (2a) it is described as very large/small; in (2b) Prof. Jones worries that it will be too large/small for some relevant purposes; and so forth (I return to (2d) below). This can again be represented schematically, as in (6):




Thus the analysis of *many* and *few* that is required for the complex quantifiers in (3) can also be extended to the simpler constructions in (1) and (2).

Based on this, my central claim is that ***many* and *few* (in positive form) denote predicates of scalar intervals** (see Schwarzschild 2006 and Rett 2006 for a similar view).

To formalize this insight, I introduce a scale *S* composed of a set of degrees *d* (i.e., the natural numbers) ordered by the 'greater than' relationship (>). I further introduce the notion of an **interval**, a set of contiguous degrees defined formally as in (7) (here I do not take the interval to be a primitive type, but rather use this term as a notational convenience to describe a certain subset of the domain of elements of type ⟨dt⟩):

(7) A set of degrees $I \in D_{\langle dt \rangle}$ is an **interval** iff
$\forall d, d', d'' \in D_d$ such that $d > d'' > d'$, $(d \in I \wedge d' \in I) \rightarrow d'' \in I$

I then propose that in their basic lexical forms, *many/few* express relationships between degrees and intervals, with *few* defined as the negation of *many* (cf. Heim's 2006 analysis of *little* as degree negation):

(8) a. $\|\text{many}\| = \lambda d \lambda I_{\langle dt \rangle}.d \in I$ b. $\|\text{few}\| = \lambda d \lambda I_{\langle dt \rangle}.\neg (d \in I)$

On these definitions, *many* and *few* do not themselves introduce the contextually relevant standard of comparison that is invoked when they occur in positive form. To achieve this, I follow a long tradition (Cresswell 1977; von Stechow 1984, 2006; Kennedy 2001, 2006; Heim 2006) in proposing that the semantics of the positive forms are derived via combination of the entries in (8) with a null positive morpheme POS. In particular, I follow the

approach of von Stechow (2006) and Heim (2006) in taking POS to introduce a contextually defined 'neutral range' $N_S$ on the scale $S$ that consists of those values that would be considered neither large nor small in the given context. Then (9) gives the semantics of *many* and *few* in their positive forms, after combination with POS. (For the present, I assume that POS composes locally with *many/few*, a point that I will return to below.)

(9) a. $\llbracket \text{POS-many} \rrbracket = \lambda I_{\langle dt \rangle}.\forall d \in N_S[d \in I]$ b. $\llbracket \text{POS-few} \rrbracket = \lambda I_{\langle dt \rangle}.\forall d \in N_S[\neg(d \in I)]$

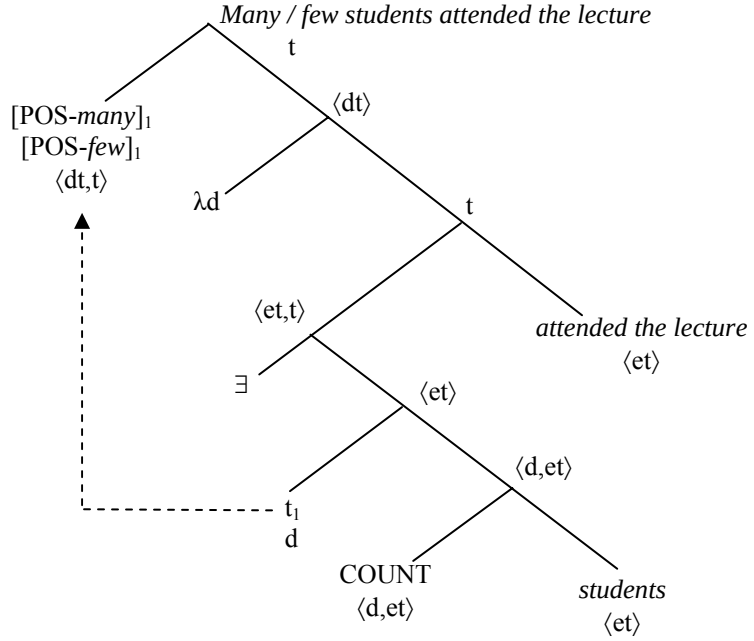
To paraphrase, POS-*many* is true of an interval $I$ if $N_S$ is contained within $I$; POS-*few* is true of an interval $I$ if $N_S$ is contained within the complement of $I$.

As defined here, *many* and *few* are not inherently quantificational, and do not themselves contribute a degree argument to the logical form. Following Schwarzschild (2006) and Rett (2006), I take the degree argument to be introduced by a covert measure function COUNT, which is introduced by a functional head within the DP, in whose specifier position *many* and *few* originate. The semantics of COUNT are given in (10) (where X is a variable that ranges over groups or plural individuals):

(10) $\llbracket \text{COUNT} \rrbracket = \lambda d\, \lambda X.\text{COUNT}(X) = d$

I furthermore take quantification over individuals to arise via existential closure. With this in place, the sentences in (1) have the derivation shown schematically in (11). Note here that POS-*many*/POS *few* originate within the DP, but raise for purposes of interpretability to take sentential scope.

(11)




The resulting structure has the interpretation given (first formally, then more simply) in (12).

(12) a. $\llbracket 1a \rrbracket$: Many students attended the lecture
[POS-many[λd.d-COUNT students attended the lecture]]
$\forall d \in N_S\, [\exists X.*\text{student}(X) \wedge \text{AtL}(X) \wedge \text{COUNT}(X) = d]$
$N_S \subseteq$ **[0, #-Students-AtL]**

b. $\llbracket 1b \rrbracket$: Few students attended the lecture
[POS-few[λd.d-COUNT students attended the lecture]]
$\forall d \in N_S\, [\neg\exists X.*\text{student}(X) \wedge \text{AtL}(X) \wedge \text{COUNT}(X) = d]$
$N_S \subseteq$ **(#-Students-AtL, ∞]**

To paraphrase, (1a) is true if the neutral range $N_S$ is fully contained within the interval from 0 to the number of students attending the lecture, or equivalently, if the number of students attending the lecture exceeds the neutral range. Conversely, (1b) is true if $N_S$ is fully excluded from that interval, i.e., if the number of students attending falls short of the neutral range.

I turn now to the complex quantifiers in (3). Since under the present proposal *many* and *few* are predicates of intervals, the comparatives in (3a-c) must provide intervals to serve as their arguments. To account for this, I propose that comparative morphology introduces a function from intervals to intervals (von Stechow 1984; Kennedy 2001). For example, in (3a), *-er than 100* maps the interval **[0, #-Students-AtL]** to the interval **[0, #-Students-AtL – 100]**. This may be formalized as follows, where ZERO is a function that maps an interval to a second interval of equal length whose lower bound is zero (Kennedy 2001):

(13) $\|$-er than d$\|$ = $\lambda d \lambda I_{\langle dt \rangle}$.[ZERO(I – **[0, d]**) if d ∈ I; ∅ otherwise]

With this interpretation, *-er than d*, like *many* and *few* themselves, must raise to take sentential scope for purposes of interpretability. Then (3a) has the logical form and resulting semantic interpretation given in (14) (where it is assumed that the 'neutral range' introduced here may be different from that in (12) above):

(14) Many more than 100 students attended the lecture
[POS-many[-er than 100 [λd.d-many[λd'.d'-COUNT students attended the lecture]]]]
100 ∈ **[0, #-Student-AtL**] ∧ $N_S$ ⊆ ZERO(**[100, #-Student-AtL]**)

The entry in (13) likewise yields correct results for *few more than 100* / *many fewer than 100* in (3b,c), with the assumption that in the latter case the presence of *few* triggers the corresponding negation of the interval **[0, n]**. Thus (3c) receives the semantic interpretation in (15):

(15) Many fewer than 100 students attended the lecture
100 ∈ **(#-Student-AtL, ∞**] ∧ $N_S$ ⊆ ZERO(**[#-Student-AtL, 100]**)

Finally, consider again the unmodified comparatives in (2d). With the semantics introduced in (13) for the comparative form, it would seem that in these cases we have an argument of interval type (type ⟨dt⟩) that is left unsaturated. I propose that here this argument is existentially bound by an existential operator that picks out a non-trivial interval, i.e. an interval that does not consist of a point (Schwarzschild & Wilkinson 2002). Thus (2d) – with *more than 100* – has the logical form and semantic interpretation given in (16):

(16) More than 100 students attended the lecture
[∃I [-er than 100 [λd.d-many[λd'.d'-COUNT students attended the lecture]]]]
100 ∈ **[0, #-Student-AtL**] ∧ ∃I. I ⊆ ZERO(**[100, #-Student-AtL]**)

**4. Consequences for Degree Modification**

It has been shown here that to account for their full distribution, *many* and *few* must be analyzed as gradable predicates of scalar intervals. In this, they are in one respect similar to gradable adjectives, in that they take a degree argument that can be saturated or bound by a degree modifier. In another respect, however, the two cases are different, since the second argument of *many/few* is an interval (i.e., a set of degrees), while that of gradable adjectives is an individual:

(8) a. $\|$many$\|$ = $\lambda d \lambda I_{\langle dt \rangle}$.d ∈ I  b. $\|$few$\|$ = $\lambda d \lambda I_{\langle dt \rangle}$.¬ (d ∈ I)

(17) $\|$tall$\|$ = $\lambda d \lambda x_e$.x is d-tall

This latter point has implications for the semantics of degree modifiers: Since under this analysis *many* and *few* are of a different semantic type than gradable adjectives, we would not necessarily predict that they would allow modification by the same elements. But in fact they do. In this section, I consider the reasons for this.

In some cases it is the semantics of the degree modifiers themselves that allows for their occurrence with both adjectival (*tall*) and quantificational (*many/few*) expressions. This is true in particular for the comparative. Under the present analysis (as in other recent accounts), the comparative form raises to sentential scope for purposes of interpretability, and then takes as an argument the set of degrees created by lambda abstraction over a variable of type d left in its base position. The semantic type of the element that originally introduces the degree variable is thus irrelevant. As a result, the semantics given by (13) for *–er than 100* can translate directly to adjectival comparatives such as *-er than 6ft* in *taller than 6ft*. Thus a sentence such as (18) has the logical form and semantic interpretation in (19), directly parallel to that in (16) above.

(18) Fred is taller than 6 feet

(19) a. [∃I [-er than 6 feet [λd.Fred is d-tall]]]
b. 6ft ∈ **[0, Fred's height]** ∧ ∃I. I ⊆ ZERO(**[6ft, Fred's height]**)

A similar point can be made about the positive morpheme POS, if we follow Heim (2006) in proposing that rather than combining locally with *many/few* (as depicted in (11) and (12)), it too is a predicate of sets of degrees, which like comparative morphology must raise for interpretability. This treatment can further be extended to *very*, which can be analyzed as a version of POS that introduces a symmetrically larger neutral zone $N_S^+$ (Heim 2006; von Stechow 2006):

(20) a. $\|\text{POS}_{\langle dt,t\rangle}\| = \lambda I_{\langle dt\rangle}.\forall d \in N_S[d \in I]$
b. $\|\text{very}_{\langle dt,t\rangle}\| = \lambda I_{\langle dt\rangle}.\forall d \in N_S{+}[d \in I]$

However, other facts suggest that the representations in (20) are not the only possibilities; rather, POS and *very* at least must have interpretations that allow them to combine locally (i.e., without raising) with *many* and especially *few*.

The relevant facts are those in (21):

(21) a. A few students attended the lecture
b. A very few students attended the lecture
c. *A fewer than 100 students attended the lecture

As seen here, *few* and *very few* may occur following *a*, but comparatives such as *fewer than 100* may not.

In previous work (Solt 2006) I have argued that the sequence *a few* is derived compositionally from *few* (see also Kayne 2005 for a similar point), and have further proposed that the difference in interpretation between *few* and *a few* can be attributed to a difference in scope: While *few* involves a negative element that takes sentential scope, above the existential quantifier (as in (11) and (12) above), in the case of *a few* that same element has lower scope, below the existential operator.

This latter claim is supported by several points. First, sentences formed with *few* and *a few* can be given paraphrases that differ only in the scope of negation:

(22) a. Few students attended the lecture
'it is not the case that there was a large group of students who attended the lecture'

b. A few students attended the lecture
'there was a not-large group of students who attended the lecture'

Secondly, *few*, but not *a few*, exhibits properties characteristic of sentential negation, such as appearance with *either* rather than *too* tags (Klima 1964):

| | | |
|---|---|---|
| (23) a. | No men like Fred, and no women do *too/✓either | NEGATIVE |
| b. | Few men like Fred, and few women do, *too/✓either | NEGATIVE |
| c. | A few men like Fred, and a few women do, ✓too/*either | POSITIVE |
| d. | Many men like Fred, and many women do, ✓too/*either | POSITIVE |

Finally, *few* licenses negative polarity items, while *a few* does not:

(24) a. Few students have <u>ever</u> been to one of Prof. Jones' lectures
b. *A few students have <u>ever</u> been to one of Prof. Jones' lectures

These differences can all be accounted for with an analysis under which the negative expression *few* in *a few* does not raise to take sentential scope (as *few* does in sentences such as (1b)), but rather is interpreted in its base position, below the existential quantifier.

Under this view, we can then give (21a,b) logical forms along the lines of (25):

(25) a. A few students attended the lecture
$\exists I \exists d \exists X[\text{POS-few}(I) \wedge d \in I \wedge {*}\text{student}(X) \wedge \text{AtL}(X) \wedge \text{COUNT}(X) = d]$

b. A very few students attended the lecture
$\exists I \exists d \exists X[\text{very-few}(I) \wedge d \in I \wedge {*}\text{student}(X) \wedge \text{AtL}(X) \wedge \text{COUNT}(X) = d]$

Since *few* in (25) has *in situ* scope, it is reasonable to suspect that POS and *very* do as well. Furthermore, and crucially, the contrast in grammaticality in (21) can then be accounted for with a proposal that POS and *very* can compose directly with *few*, while *–er than 100* cannot, but rather must raise for interpretability, a possibility which is blocked by the presence of *a*.

But what sort of elements could the positive morpheme POS and its counterpart *very* be, to permit them to combine directly with a predicate of intervals such as *few*? Kennedy (to appear) introduces a version of POS that takes a gradable adjective as its argument and returns a (non-gradable) predicate of individuals. But extending this approach to the present case would require that POS have different semantics in the adjectival and quantificational domains, given that the two classes of expressions themselves are of different types. This would be an unwelcome result, losing much of the elegance of previous degree-bases analyses. A simpler possibility is that in addition to the type ⟨⟨dt⟩,t⟩ denotations shown in (20), POS and *very* also have interpretations at type d, allowing them to directly saturate the degree argument of *few*:

(26) $\|\mathrm{POS_d}\| = \mathrm{d}^{\mathrm{Standard}}$

Such a duality would not be surprising, given that it mirrors exactly the availability of both referential (type e) and quantificational (type ⟨et⟩,t⟩) interpretations for DPs (Partee 1987). Further support comes from the fact that while *few* is context dependent, *a few* is not. Thus the truth or falsity of (1b) depends not just on the number of students attending, but on some contextually determined notion of what would be considered a large or small number; by contrast, 3-4 students attending would be sufficient to guarantee the truth of (21a), regardless of the context. This is consistent with a proposal that in (1b) POS has the context-dependent interpretation in (20), while in (21a) it has the context-insensitive interpretation in (26). This appears to be a promising avenue to pursue, though many questions remain for further investigation, in particular the distribution of the 'referential' POS in (26), and the precise nature of the 'standard' degree that it references.

**5. Conclusions**

In this paper, I have used facts relating to the occurrence of *many* and *few* in complex quantifiers to support an analysis of these two terms as gradable predicates of scalar intervals, rather than the quantificational determiners of GQ theory. I have further shown that this analysis has implications for the semantics of the degree modifiers that combine with both *many*/*few* and gradable adjectives, suggesting that in some cases they may require interpretations of multiple semantic types, similar to what we find with DP interpretations.

**References**


Barwise, J. and Cooper, R. (1981). Generalized quantifiers and natural languages. *Linguistics and Philosophy*, 4, 159-219.

Cresswell, M. J. (1977). The semantics of degree. In B. Partee (ed.), *Montague grammar*, pp. 261–292. New York: Academic Press.

Hackl, M. (2000). *Comparative quantifiers*. PhD dissertation, MIT.

Hackl, M. (2006). On the grammar and processing of proportional quantifiers: Most versus more than half. Unpublished ms., Pomona College.

Heim, I. (2000). Degree operators and scope. *Proceedings of SALT X*, Ithaca, NY: CLC Publications.

Heim, I. (2006). Little. *Proceedings of SALT XVI*, Ithaca, NY: CLC Publications.

Kayne, R. S. (2005). On the syntax of quantity in English. In *Movement and Silence*. New York: Oxford University Press.

Kennedy, C. (2001). Polar opposition and the ontology of 'degrees', *Linguistics and Philosophy*, 24, 1, 33-70.

Kennedy, C. (to appear). Vagueness and grammar: The semantics of relative and absolute gradable predicates. To appear in *Linguistics and Philosophy*.

Kennedy, C. and McNally, L. (2005). The syntax and semantics of multiple degree modification in English. *Proceedings of the HPSG05 Conference*. Stanford, CA: CSLI Publications.

Klima, E.S. (1964). Negation in English. In J.A. Fodor & J.J. Katz (eds.), *The structure of language*. Englewood Cliffs, NJ: Prentice-Hall.

Partee, B. H. (1986). Noun phrase interpretation and type-shifting principles. In J. Groenendijk, D. de Jongh, and M. Stokhof (eds.), *Studies in discourse representation theory and the theory of generalized quantifiers*. Dordrecht: Foris.

Rett, J. (2006). How *many* maximalizes in the Balkan Sprachbund. *Proceedings of SALT XVI*. Ithaca, NY: CLC Publications.

Schwarzschild, R. (2006). The role of dimensions in the syntax of noun phrases. *Syntax*, 9, 67-110.

Schwarzschild, R. and Wilkinson, K. (2002). Quantifiers in comparatives: A semantics of degree based on intervals. *Natural Language Semantics*, 10, 1-41.

Solt, S. (2006). Why *a few*? And why not **a many*? *Proceedings of Sinn und Bedeutung 10. ZAS Working Papers in Linguistics*, 44, 6, 333-346.

von Stechow, A. (1984). Comparing semantic theories of comparison. *Journal of Semantics*, 3, 1-77.

von Stechow, A. (2006). Times as degrees: früh(er) 'early(er)', spät(er) 'late(r)', and phrase adverbs. Unpublished ms., Tübingen.