On measurement and quantification: The case of *most* and *more than half*

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Abstract

This paper presents corpus data illustrating a range of previously unnoticed distributional and interpretive differences between the superficially equivalent quantifiers *most* and *more than half*. It is shown that all of them derive from a basic difference in logical form (per Hackl 2009), which places different requirements on the structure of the underlying measurement scale: *more than half* must be interpreted relative to a ratio scale, while *most* can be interpreted relative to an ordinal or semi-ordered scale, the latter parallel in structure to the leading psychological model of humans' approximate numerical abilities. These results demonstrate the relevance of measurement theoretic concepts to natural language meaning, and point in particular to the conclusion that the grammar includes measurement scales whose points are not totally ordered with respect to one another. This case study also illustrates a close link between the semantics of quantitificational expressions on the one hand and the cognitive representation of quantity and measure on the other.

1 Introduction

1.1 Modes of comparison

Suppose I have two objects – rocks, let's say – and I want to know whether the first is heavier than the second. There are (at least) two ways that I might go about finding this out. To take one approach, I could place the first rock on a scale and write down its measured weight, do the same with the second rock, and then compare the two values I have recorded (e.g., 452 grams vs. 319 grams). Alternately, I could place the two rocks on the two pans of a balance scale, and observe which side hangs lower, without in any way representing or recording the weight of either rock individually. We might call these two procedures 'digital' and 'analog', respectively.

Now suppose that the question is whether the first rock weighs more than half as much as the second. In this case, the digital procedure can still be applied: I weigh the first rock and record that value, weigh the second rock and divide the

resulting value by two, and then compare the two values. But in this case, the analog procedure will not work: "half as much as the second rock weighs" is not an object that can be placed on a balance.

There is a fundamental difference between the two modes of comparison illustrated above. The first assigns each entity a numerical measure which can be compared to the measures of other individuals, and which may serve as input to mathematical operations (in the example above, division by two). The second, on the other hand, merely establishes a relationship of 'greater than' between two entities. Given some set of entities, we could via a series of pairwise comparisons establish a rank ordering of its members. This ordering could in turn be given a numerical representation in an order-preserving way, for example by assigning the number 1 to the highest ranked entity, 2 to the next highest ranked, 3 to the third highest ranked, and so forth. But the numbers assigned in this way would in an important sense be arbitrary. By contrast, in the case of the first procedure, the only arbitrariness is in the choice of the unit of measurement; once this is fixed, so too is the value assigned to each individual.

These observations are by no means new. There is an entire field of study, namely measurement theory, devoted to understanding how the properties of and relationships between entities can be represented numerically (Kranz et al. 1971). A basic finding is that not all types of relationships can be given a numerical representation that supports comparisons such as that between the ratios of measures. Results from cognitive psychology support a somewhat parallel distinction in how humans perform comparisons: in particular, comparison of the sizes of two sets may proceed via a precise symbolic representation of their cardinality, or via a more basic analog representation of their magnitude (Dehaene 1997).

The main theme of the present paper is that distinctions of this sort are relevant to language as well. Specifically, certain expressions of measure assume the possibility, at least in principle, of applying one or the other of these procedures to their assessment. This distinction has consequences for both the distribution and the interpretation of these items.

The domain of inquiry within which I will explore this topic is quantification, with specific focus on the proportional quantifiers *most* and *more than half*, as in the following:

(1) a. Most Americans have broadband internet access
    b. More than half of Americans have broadband internet access

The specific claim that I will argue for is that *more than half* assumes the first of the two modes of comparison discussed above (the digital one), while *most* favors

the second (the analog one).

More broadly, the results from this domain will provide evidence for the relevance of measurement theoretic concepts to natural language meaning. Specifically, measurement scales vary in the strength of the ordering relationships that they are based on, and this has linguistic consequences. Beyond this, this case study will point to a meaningful connection between the semantics of quantificational expressions on the one hand and the cognitive representation of quantity and measure on the other.

1.2 The semantics of *most* and *more than half*

At first glance, (1a) and (1b) seem to be more or less equivalent in meaning, both true if the relationship in (2) holds:

(2) # of Americans who have BB $>$ # of Americans who don't have BB

In fact, introductory semantics textbooks (e.g. Chierchia and McConnell-Ginet 2000) typically present the semantics of *most* as yielding the truth conditions in (2).

That such an analysis is too simplistic has been pointed out by many authors (see e.g. Huddleston and Pullum 2002). Most obviously, speakers commonly have the intuition that *most* and *more than half* differ in their lower bounds: while a simple majority is sufficient to establish the truth of an examples such as (1b) with *more than half*, a greater proportion is required for the corresponding *most* example (e.g. (1a)). This is illustrated nicely by examples such as the following:

(3) a. ??Most of the American population is female
b. More than half of the American population is female

In the situation where the American population has a very slight female skew (in 2009, the numbers were 50.7% female vs. 49.3% male), (3b) is a true statement, while (3a) is highly infelicitous. Speakers do not entirely agree as to the nature of the infelicity: some judge (3a) as outright false, while others feel it to be true but pragmatically inappropriate. But there is no disagreement that there is a sharp contrast here.

Contrasts in interpretation between *most* and *more than half* have also figured prominently in the debate on the semantics/pragmatics interface. *More than half* is unarguably defined by its lower bound. On the neo-Gricean view championed in particular by Horn (2005), *most* is likewise semantically lower bounded, with the upper bound ('not all') derived pragmatically via scalar implicature. An alternate

view is offered by Ariel (2004, 2005), who proposes that the lexical meaning of *most* provides an upper as well as a lower bound (while still allowing *most* to be compatible with the situation in which 'all' obtains).

In more recent work, which is particularly relevant to the discussion to come, Hackl (2009) proposes that both *most* and *more than half* require compositional analyses; the distinct logical forms that are so derived are truth conditionally equivalent but give rise to different verification procedures. Hackl supports this claim with experiments using a 'self-paced counting' paradigm. I will return to a more in depth discussion of Hackl's proposal below.

1.3 The present study

In this paper, I take a novel look at the semantics of *most* and *more than half*. Drawing on corpus data illustrating their use in American English, I demonstrate previously unnoticed differences in their distribution, which I will argue (building on Hackl's analysis) stem from a fundamental difference in logical form that corresponds to a distinction in what sort of underlying ordering relationship assumed, a distinction that mirrors that between the two weighing procedures discussed at the start of the paper.

The organization of the paper is the following. In Section 2, I present the corpus data. Section 3 introduces the proposal, which draws on concepts from measurement theory and from cognitive psychology to formalize the weighing analogy discussed above. Sections 4 and 5 work out in more depth how the analysis can account for the observed patterns in the distribution of *more than half* and *most*, respectively. Finally, Section 6 summarizes the conclusions, and draws parallels to related phenomena, in particular the divergent behavior of the positive and comparative forms of gradable adjectives.

2 Corpus analysis

2.1 Data source

Data for the analysis were drawn from the Corpus of Contemporary American English (COCA) (Davies 2008-), a 425 million word (approximately 20 million words per year for the years 1990-2011) corpus equally divided among spoken

language, fiction, popular magazines, newspapers, and academic texts.[1]

In total, there are 432,830 occurrences of *most* and 4857 occurrences of *more than half* in COCA. However, not all of these are relevant to the present analysis. In what follows, I focus on the use of these expressions as quantifiers in the nominal domain. I put aside the so-called relative proportional use of *most* (4a), as well as its use in adjectival superlatives (4b). Presumably there is a relation between the semantics of *most* in these uses and in its quantificational use (cf. Hackl 2009, for an analysis that links quantificational and relative proportional *most*), but I will not attempt to make the connection here. I also do not consider the use of *more than half* as an adverbial modifier (5), though again one suspects that a connection could be drawn.

(4) a. Anabel read the most books
    b. Anabel is the most diligent student in the class

(5) The project is more than half finished

2.2 Findings

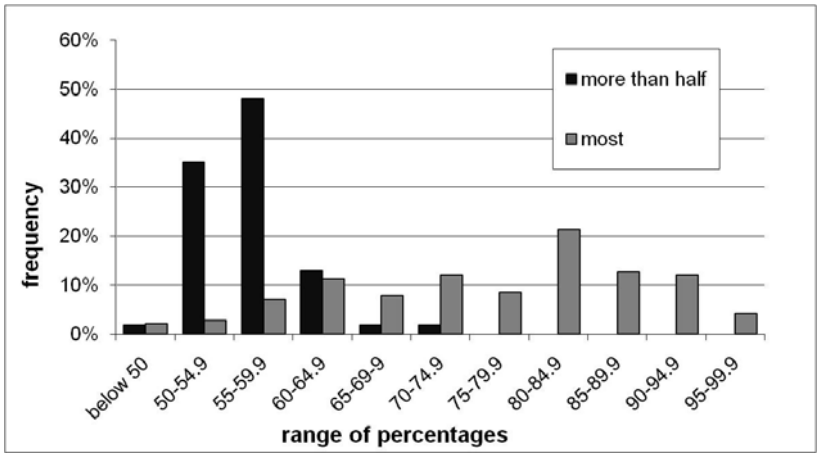
A look at the corpus data demonstrates deep distributional and interpretive differences between *most* and *more than half.*

Percentage range. First, the corpus data substantiate the intuition that *most* is associated with higher proportions than *more than half.* Of course, in the majority of corpus examples involving the use of a quantifier, it is not possible to determine what the actual proportion is. However, there is one particular type of example, found quite commonly in the reporting of survey data, where a quantifier is used in conjunction with an exact percent; this usage gives us precisely the sort of data we need to investigate the proportions which are described by the two quantifiers in question. In (6) and (7) we see typical examples of this sort for *most* and *more than half*, respectively. Note that the cited percentages are considerably higher in the former than the latter case.

(6) a. The survey showed that most students (81.5%) do not use websites for math-related assignments (Education, 129(1), pp. 56-79, 2008)

[1]Data for the analysis were extracted in mid-2009, when the corpus covered the years 1990-2008, and its size was approximately 385 million words

Figure 1: Percentages associated with *most* and *more than half*



b. Most respondents – 63 percent – said the best movie for date night is a comedy (Redbook, 208(6), p. 158, 2007)

c. Most Caucasian grandparents were married (67%), had attained an education level above high school (64%), and lived on an annual household income above $20,000 (74%). (Journal of Instructional Psychology, 24(2), p. 119, 1997)

d. Most respondents (92.6 percent) had completed high school (Journal of Environmental Health, 69(5), pp. 26-30, 2006)

(7) a. More than half of respondents (55%) say that making money is more important now than it was five years ago (Money, 21(3), p. 72, 1992)

b. More than half of the respondents (60%) earned Ph.D. degrees (Physical Educator, 53(4), p. 170, 1996)

c. And while more than half of us grill year-round (57 percent), summertime is overwhelmingly charcoal time (Denver Post, 24/5/2000)

d. Booz Allen Hamilton, a technology consultancy, concluded in a study that more than half of new hires (51 per cent) were found through the Internet (Christian Science Monitor, 2000)

This pattern is confirmed quantitatively. Figure 1 tallies the 54 examples of this type found in the corpus for *more than half*, and the corresponding 141 exam-

ples, involving the same nouns, for *most*.[2] These data show that *more than half* is almost exclusively used for percentages between fifty and sixty-five percent, while *most* is rarely used for percentages below sixty percent, and quite common up to ninety percent and higher. This pattern to a large extent mirrors results of acceptability judgments reported by Ariel (2004), which show that that *more than half* is accepted at a higher rate than *most* for proportions close to 50%.[3]

Generic vs. survey results readings. The numerical data discussed above are the first indication of a divergence in the behavior of *most* and *more than half*, but the differences go much deeper than this. As seen in the corpus examples in (8), *most* can readily be followed directly by a plural noun phrase, and in this case tends to take on a generic interpretation. (8a), for example, says something about the behavior of people in general; (8c) tells us something about teens in general.

(8) a. Most people follow the moral judgments of those around them. (Writer, 121(7), pp. 30-33, 2008)
b. Money is at least partly a control issue in most families. (Money, 32(1), p. 106, 2003)
c. Most teens want to fit in with their peers. (CNN YourHealth, 31/8/2002)

By contrast, *more than half* has a very different feel. As illustration, when *most* in natural examples of this sort is replaced by *more than half of*[4], as in (9), the results are decidedly odd, and the generic flavor is lost entirely. Rather, to the extent that the resulting sentences are acceptable, they have what might be termed a 'survey results' interpretation; (9c), for example, seems to report on some sort of survey of teens.

(9) a. ?More than half of people follow the moral judgments of those around them.
b. ?Money is at least partly a control issue in more than half of families.
c. ?More than half of teens want to fit in with their peers.

[2]The noun was held constant in order to avoid any possible confound that might arise if particular nouns tend to be used with particular percentage ranges.

[3]In fact, Ariel's data are for the Hebrew word *rov* 'most' rather than the English *most*; however, absent evidence to the contrary it is reasonable to assume their interpretations to be parallel.

[4]*More than half* cannot be followed directly by a plural noun, i.e. **more than half people*, but instead occurs in a pseudopartitive structure with *of*.

Noun phrase structure. The preceding examples point to both syntactic and interpretive differences between noun phrases containing *most* and *more than half*. Both of these can be characterized in greater depth. To start with the syntactic side, the felicitous examples for *most* in (8) involved the quantifier being followed directly by a plural nominal. In the case of *most*, this is common. But by contrast, it is quite infrequent that *more than half* occurs in a pseudopartitive construction with a bare plural nominal. Instead, the majority of occurrences involve a partitive construction, where *more than half* is followed by a definite description (10a,b) or pronoun (10c), or by a noun phrase of the form *all NP* (10d).

(10) a. . . . more than half of the men detained at Guantanamo have never committed a single act of hostility against the U.S. government. (Denver & the West, letters to the editor, 1/18/2007)

b. Altfest says that more than half of his clients who accept early-out offers continue to work, often as consultants. (Fortune, 143(2), p. 247, 2001)

c. In Illinois, the organ bank reports nearly 4,000 people on the organ transplant waiting list – more than half of them waiting for kidneys. (Chicago Sun Times, 11/8/1999)

d. More than half of all farmworkers earn less than $12,500 annually (Ms, 15(2), p. 40, 2005)

To quantify this distinction, a sample of occurrences of the string "more than half of" was extracted from the corpus as follows: for each of the years 1990-2008, and each of the five genres included in the corpus (newspaper, magazine, fiction, academic and spoken), ten tokens of *more than half of* were extracted, and classified according to the structure of the following nominal element. The same was done for *most*. Instances of relative proportional *most* and of *most/more than half* as an adverbial modifier were excluded.

The results are shown in Table 1. As seen here, the majority of *more than half* tokens (70%) involve a definite description, with plural noun phrases accounting for just a minority of cases (8%). By contrast, *most* occurs in a wider range of constructions; it is most commonly (57% of tokens) followed directly by a plural nominal, but definite descriptions and pronouns are also common (31% and 10% of tokens, respectively).

Kind vs. group nominals. The difference in the noun phrase structures found with *most* and *more than half* corresponds to an interpretive difference. Consider

Table 1: Following nominal

	Most		More than half	
	#	%	#	%
Definite description	282	31	606	70
Pronoun	95	10	79	9
Bare plural	516	57	68	8
'All NP'	0	0	72	8
Bare singular	8	1	17	2
Other	8	1	18	2
Total	909	100	860	100

the *more than half* examples in (10). What unifies cases involving definite descriptions and pronouns, as in (10a-c), is that the downstairs noun phrase denotes a set or group of individuals localized in space and time (e.g. 'the men detained at Guantanamo'; 'his clients who accept early-out offers'; 'them', referring to patients on Illinois' transplant list). The same can be said about examples with *all*, such as (10d): here, it is the maximal (contextually relevant) set of farmworkers over which *more than half* quantifies.[5] By contrast, the plural noun phrases in the *most* examples in (8) are most easily interpreted as denoting the members of a kind in general.

Furthermore, this generalization extends to those minority of cases where *more than half* is followed directly by a plural noun phrase. While an example such as (9c) is odd, and particularly cannot be read as saying something about teens generally, (11) is fine; the difference is that here, it is a particular group of teens (those who took part in the mentioned survey) whose attitudes are characterized.

(11) More than half of teens surveyed said they are "not too careful or not at all careful" to protect their skin. (Todays Parent, 23(7), p. 154, 2006)

In what follows I will use the terms 'kind nominal' and 'group nominal' to refer to these two types of plural expressions (though without claiming that the nominal in the kind cases actually denotes the kind, nor taking any position as to what sort of thing a kind is).

[5]In this respect, the precise function of *all* in examples such as (10d) is not entirely clear. For myself, the corresponding example without *all* remains grammatical, though less natural, and there is no obvious change in meaning. Brisson's (2003) of *all* as a maximizing element may be relevant here.

Table 2: Presence of supporting data

	Cases with supporting data mentioned	
	Most	More than half
Americans	13/100	5/12
Men	5/100	4/6
Women	7/100	4/5
Students	36/100	5/5
Patients	39/100	5/5
Families	11/100	1/2
TOTAL	111/600	28/35
	19%	80%

It is modification that makes the group reading salient. Unmodified *teens* in (9c) is most easily interpreted as denoting teens as a whole, or some unspecified subset thereof, giving the sentence as a whole a generic feel. By contrast, the modified *teens surveyed* denotes a particular set of members of that kind. This pattern can also be confirmed quantitatively: in 75% of tokens of *more than half* in a pseudopartitive construction, the noun is modified in some way (e.g. with an adjective or relative clause); this compares to just 38% of tokens for *most*+plural.

Supporting data. The intuition that *more than half* (in contrast to *most*) has a 'survey results' interpretation is corroborated by examining the degree to which some sort of data is cited to support the use of the two quantifiers. To assess this, six plural nouns were selected that occurred multiple times in the corpus with *more than half*; for each, the proportion of cases was tallied in which there was mention of supporting data in the immediate context. As supporting data, I counted explicit numerical data (as in examples such as (6) and (7)), mention of a study or analysis, or a footnote to a data source. For comparison, the same analysis was done for the first 100 occurrences of each of the the same six nouns with *most*.[6] As seen in Table 2, the difference is dramatic: in 80% of cases with *more than half*, there is some reference to supporting data; but this is the case for just 19% of the cases with *most*. To put this diffently, *more than half* is typically used when actual numerical data is being reported; *most* is not.

[6]The restriction to 100 tokens per noun was necessitated by the very large number of occurrences of *most* with these nouns, e.g. over 2000 occurrences of *most Americans*.

Vague and uncountable domains. Perhaps the most intriguing difference between *most* and *more than half* relates to the sort of domains over which they quantify. Consider corpus examples such as the following (which are representative of many similar cases):

(12) a. But like most things, obesity is not spread equally across social classes (Mens Health, 23(7), p. 164, 2008)
b. Most beliefs, worries, and memories also operate outside awareness (Science News, 142(16), 1992)
c. But he had enough material on his truck to handle most problems (Contractor, 47(4), p. 30, 2000)

All of these sentences are entirely felicitous, but when one stops to think about it, there is something quite odd here. To take (12a) as an example, the 'things' whose distribution across social classes is discussed do not seem to be the sort of entities that we could put on a list and count. The same could be said about 'beliefs, worries and memories' and 'problems' contractors face: what constitutes an individual member of that domain is vague. (For example, what exactly constitutes a 'problem' experienced by a contractor? When does one end and the next start?) Yet quantification with *most* is nonetheless acceptable.

The same cannot be said about *more than half*. When *most* in the attested examples is replace *more than half*, as in (13), the result is once again peculiar:

(13) a. ?But like more than half of things, obesity is not spread equally across social classes.
b. ?More than half of beliefs, worries, and memories also operate outside awareness.
c. ?But he had enough material on his truck to handle more than half of problems.

The infelicity of the examples in (13) is not merely due to the above-mentioned tendency for *more than half* to occur in a partitive construction; the corresponding examples with a partitive or *all* NP are also peculiar:

(14) a. ?But like more than half of all things, obesity is not spread equally across social classes.
b. ?More than half of the patient's beliefs, worries, and memories also operate outside awareness.

c. ?But he had enough material on his truck to handle more than half of the problems contractors face.

Rather, the source of the oddness is the implication of enumerability that results: (13a), for example, seems to imply that we have in fact made an list of exhaustive list of 'things', and gone down that list to count how many are spread equally across social classes.

The following pair provides further support that lack of specificity is not the culprit. The denotation of *neighborhoods in Port-au-Prince* is localized in space (Port-au-Prince, Haiti) and time (here, the time of writing). Yet what constitutes a neighborhood is, in everyday speech, only loosely defined: boundaries between neighborhoods are informal at best, and there is no hard and fast rule for deciding whether a particular area of a city is a single neighborhood or two or more (or not part of a neighborhood at all). This indeterminateness does not affect quantification with *most* (15a), but when *more than half* is substituted (15b) it has the odd effect of suggesting that there is some exhaustive list of neighborhoods in Port-au-Prince. That is, the noun phrase here must be coerced to an enumerable interpretation to allow quantification with *more than half*.

(15) a. Most neighborhoods in Port-au-Prince have community organizations that were in place prior to the earthquake. (Christian Century, 127(7), p. 10, 4/6/2010)

b. ?More than half of neighborhoods in Port-au-Prince have community organizations that were in place prior to the earthquake.

Thus *more than half* (but not *most*) requires a domain whose members can, at least in principle, be individuated (separated from one another in a non-arbitrary way) and enumerated (put on an exhaustive list and counted).

To substantiate these observations quantitatively, all of the tokens of *more than half* followed by a plural nominal, and a corresponding sample of tokens of *most* in this context, were classified according to the denotation of the nominal. The categories used were: people; units of people (e.g. households, couples); things/animals/locations; organizations (e.g. companies); 'events/cases', under which category were grouped repeatable occurrences or instances of a phenomenon (examples include *accidents*, *fireworks-related injuries*, *iTunes downloads* and *diabetes and prediabetes cases*); other abstract count denotations; mass denotations (typically pluralia tanta such as *resources* and *supplies*).

The results are given in Table 3. As seen here, in fully three quarters of the *more than half* examples, the plural nominal denotes a set of people (e.g. Ameri-

Table 3: Denotation of following nominal

	Most		More than half	
	#	%	#	%
People	79	54	186	75
Units of people (e.g. couples, households)	3	2	11	4
Things/animals/places	18	12	12	5
Organizations	10	7	10	4
Events/cases	13	9	17	7
Other abstract count	24	16	0	0
Mass	0	0	11	4
TOTAL	147	100	247	100

cans, doctors, etc.), arguably the most readily countable type of entity there is (as a relevant parallel, Smith-Stark (1974) provides evidence that if a language marks number at all, it does so for nouns denoting humans). Things/animals/locations, units of people, organizations and events/cases (all fairly countable) are represented with a much lower level of frequency, and crucially, there are no instances of other types of abstract count nouns. By contrast, the category 'people' accounts for only half of the *most* tokens. The category 'other abstract count' makes up the next largest (16% of tokens). While some of these have easily countable denotations (e.g. *PhD programs*), we also find examples such as *courtroom duties*, *prevention efforts for young adults* and *steps the companies are taking*, entities that do not lend themselves to enumeration and counting.

Quantitative measurement. The examples discussed in the previous section involved plural nouns, and the dimension in question was number. A similar contrast is observed in the mass domain, where another dimension of measurement is involved. In (16) we see typical examples featuring *more than half* with a mass noun. Here, the dimensions involved (energy, area, liquid volume) can all receive a quantitative measure (expressed for example in kilowatt hours in the case of energy, square kilometers in the case of area, etc.).

(16) a. More than half of home energy use goes to space heating and cooling. (Popular Mechanics, 184(6), p. 79, 2007)

b. In 1997, non-OPEC producers accounted for more than half of world oil production. (Futurist, 33(3), p. 51, 1999)

c. In contrast to Turkey's relative abundance, more than half of Syrian territory gets less than 250 millimeters of rainfall per year. (Journal of International Affairs, 49(1), p. 123, 1995)

By contrast, *most* can occur with mass nouns whose denotations are abstract or unmeasureable, as in (17); replacing *most* with *more than half* again results in the same sort of infelicity observed in the count noun case.

(17) a. But black activists acknowledge that most racism is not so blatant. (Associated Press, 16/9/1991)

b. ?But black activists acknowledge that more than half of racism is not so blatant.

Importantly, the infelicity of the (b) example again cannot be attributed to the pseudopartitive structure or the nonspecificity of *racism*. Examples of the following sort, involving a partitive structure and a spatially and temporally bounded manifestation of the phenomenon, are likewise bad. Rather, the issue is that racism is not the sort of entity that can be given a quantitative measure.

(18) a. Most of the racism in America today is caused by ignorance.

b. ?More than half of the racism in America today is caused by ignorance.

Taken together, the examples in this section and the previous one demonstrate that *more than half* requires a domain whose members can be counted, or (in the mass case) which can otherwise be measured. When this fails – either because the individuals in the domain are too vague to allow counting, or because there is no quantitative measure – quantification with *most* is nonetheless possible, but quantification with *more than half* is not.

Vague predicates. The preceding section discussed a contrast in the domains over which *most* and *more than half* may quantify. A similar though somewhat more subtle contrast is observed in the sentential predicates that may co-occur with the two quantifiers. *Most* may occur with vague predicates, as in (19) (the predicates in question being *wealthy*, *ordinarily talented* and *lovely in their own way*):

(19) a. . . . most of the tourists in the early days were wealthy. (Natural Parks, 83(2), p. 14)

b. Most of our employees are, like me, ordinarily talented. (Fortune, 157(13), p. 129, 2008)

c. Fortunately, most of the wines in our tasting were lovely in their own way. (Chicago Tribune, 21/5/2008, p. S7)

The corresponding *more than half* examples, by contrast, are slightly odd[7]. (20a), for example, seems to subtly raise the question of what the standard for *wealthy* is, while (20b) similarly seems to imply that *ordinarily talented* has some precise criteria of application.

(20) a. ?. . . more than half of the tourists in the early days were wealthy.
b. ?More than half of our employees are, like me, ordinarily talented.
c. ?Fortunately, more than half of the wines in our tasting were lovely in their own way.

Thus *more than half*, but not *most*, requires a sentential predicate with a precisely defined extension.

2.3 Summary

The results of the corpus analysis can be summarized as follows:

1. *Most* and *more than half* are used for distinct ranges of proportions: the range of application for *more than half* begins 'sharply' at just over 50% and ends around the proportion of two thirds; *most* is rarely used near the 50% mark, but is found commonly for proportions up to 90% and above.

2. Whereas *most* tends to have a generic flavor, *more than half* has a 'survey results' interpretation, and is typically used when supported by data.

3. The distribution of *more than half* is in several respects restricted relative to that of *most*, in that *more than half*:

   (a) occurs primarily in a partitive construction, or in a pseudopartitive with a group-denoting nominal;

   (b) requires a countable or otherwise measurable domain;

   (c) requires a precisely defined predicate.

[7] I report my own judgments here. Some speakers I have consulted find no contrast between examples such as (19) and (20), both being entirely acceptable. For myself, however, the distinction is subtle but very real.

In what follows, I will show that all of these differences derive from a fundamental difference in logical form, which corresponds to a difference in the underlying ordering structure that the two quantifiers assume.

3 The proposal

3.1 Logical form and scale structure

In approaching the facts discussed in the preceding section, I take as my starting point a proposal put forward by Hackl (2009), who investigated the semantics and processing of *most* and *more than half*. Hackl argues that both quantifiers require compositional analyses. *Most* should be analyzed as the superlative form of *many* (an idea that goes back to Bresnan 1973, and much earlier work), while *more than half* should be analyzed as a comparative, incorporating the notion of half, namely division by two. The resulting logical forms, in somewhat more general form, are given below[8]:

(21) a. ⟦most⟧(A)(B) = 1 iff $\mu_S(A \cap B) > \mu_S(A - B)$
b. ⟦more than half⟧(A)(B) = 1 iff $\mu_S(A \cap B) > \mu_S(A)/2$

Here μ_S is a measure function, that is, a function that maps individuals to degrees on the scale *S*.

While (21a) and (21b) are superficially equivalent, Hackl proposes that the difference in form corresponds nonetheless to a difference in the verification procedure used in sentence processing. *Most* triggers a 'vote-counting' procedure: For each *A* that is *B*, determine whether there is at least one *A* that is not *B*; if not, the sentence is true. *More than half*, by contrast, triggers a procedure involving totaling the number of *A* that are *B*, and comparing that number to half the total number of *A*. Hackl supports this claim with experimental data. Via a novel 'self-paced counting' paradigm, he demonstrates that when presented with a task which favors a vote-counting procedure, subjects are quicker when the quantifier presented is *most* than when it is *more than half*.

[8]Hackl considers only the count noun case, and as such his logical forms are based on a set cardinality operator. But as seen in the preceding section, both *most* and *more than half* are also compatible with mass nouns (e.g. *most land in Denmark*; *more than half of home energy use*), where the dimension in question is something other than cardinality. The formulae given in (21) generalize to these cases as well.

I would like to suggest that this effect on verification times represents only the tip of the iceberg, so to speak, in the consequences from the difference in logical form. In fact, the patterns identified in the corpus data also follow from this same source.

The central claim I will argue for here is that while both the logical forms in (21) are based on a measure function μ_S, they differ in the underlying structure of the scale that serves as the range of this function, a difference that corresponds to that between the two weighing procedures that were discussed at the start of this paper.

In formalizing this idea, I assume an ontology that includes degrees as a basic type (type *d*). A scale *S* can be conceptualized as a triplet $S = \langle D, >, DIM \rangle$, where D is a set of degrees, $>$ is an ordering relationship on that set, and *DIM* is the dimension of measurement. Minimally, the structure of *S* must be such that $\mu_S(a) > \mu_S(b)$ if and only if *a* has more of the property corresponding to *DIM* (e.g. height, weight, cardinality) than does *b*. But above and beyond this, there are a range of possibilities for the structure of *S*.

I would like to argue that (21a) and (21b) place different requirements on scale structure. To return to the analogy of weighing rocks, the logical form for *most* in (21a) can be assessed with reference to a scale *S* that corresponds to the output of an analog comparison procedure, comparable to the procedure of placing two rocks on a balance scale to determine whether the 'heavier than' relationship holds between them, without establishing the weight of either one individually. But this is not sufficient to support the assessment of the logical form for *more than half* in (21b). Rather, just as in the case of verifying the 'weighs more than half as much as' relationship between objects, we require a scale *S* that supports the comparison of ratios of measures, such as that corresponding to the output of a digital weighing procedure (e.g. assigning each rock its weight in grams). Below, I translate this simple analogy into more formal terms.

3.2 Measurement theory

The distinction between the strength of the underlying ordering relationships required for *most* and *more than half* can made more precise by drawing on concepts from measurement theory (Stevens 1946; Kranz et al. 1971). Measurement, in a formal sense, consists in associating entities with numbers (or other mathematical objects) in such a way that the properties of and relationships between entities are faithfully represented by the properties of and relationships between numbers. To take a simple example, the weight of objects can be given a measure-theoretic

representation via a function φ that satisfies the condition that $\varphi(a) > \varphi(b)$ iff *a* has more weight than *b*.

It is now standard to distinguish several levels of scales that are created as the result of measurement. An **ordinal** scale represents a simple rank ordering, with no notion of distance between scale points. Examples of ordinal scales include the rank order of runners finishing a race (first, second, third, etc.), and the Mohs scale of mineral hardness, which is based on the pairwise relationship '*a* scratches *b*' (e.g. diamond scratches quartz; quartz scratches fluorite; etc.). The analog weighing procedure described at the beginning this paper would likewise give rise to an ordinal scale. Such scales support comparisons of measures (e.g. *this rock is heavier than that rock*), but nothing further. An **interval** scale adds a fixed unit of measure, and thus supports statements of magnitude of difference (e.g. *this rock is 23 grams heavier than that rock*). The classic example of an interval scale is temperature measured in Celsius or Fahrenheit. Finally, a **ratio** scale adds a non-arbitrary zero point, and thus supports the expression of ratios of measures (e.g. *this rock is twice as heavy as that rock*). Examples of physical measures that invoke ratio scales include set cardinality represented via the natural numbers, height in centimeters, and weight in grams; thus the digital weighing procedure described earlier produces a ratio scale. Ratio level measurement is sometimes called extensive measurement. The distinction between ratio and ordinal scales, the two scale types which will be most relevant below, is illustrated schematically in Figure 2a,b.

Concepts from measurement theory have been applied to semantic analysis by Krifka (1989); Klein (1991); Nerbonne (1995), among others. Recently, Sassoon (2007, 2010) has argued that contrasts in the occurrence of measure phrases (e.g. *3 feet tall* vs. **50 degrees warm*) and modifiers such as *twice* (e.g. *twice as tall* vs. ?*twice as short*) can be related to the level of measurement assumed by the adjective: positive adjectives such as *tall* are associated with ratio scales, while those such *short* and *warm* invoke only interval scales. This approach is extended by van Rooij (2010), who shows that natural language comparative constructions differ in the structure of the measurement scales they need for their evaluation: only some require ratio-level measurement, while others can be evaluated with regards to an interval or ordinal level scale.

Returning to the issue at hand, I would like to propose that a similar variation in scale structure is involved in the case of *most* and *more than half*. It is readily seen that an expression of the form in (21b) assumes a ratio scale, in that it expresses the ratio of two measures, that of $A \cap B$ and that of A. That is, the logical form of *more than half* presupposes measurement at the ratio level. There

is nothing terribly surprising here, in that counting, as well as typical measures of mass, volume, etc., represent classic examples of ratio level measurement. What is important is that in the case of (21a), a ratio scale is not required. Rather, a simple ordering of entities will suffice, one that can be represented via an ordinal scale. That is, the logical form of *most* need only assume an underlying qualitative ordering, without building in notions of distances or ratios between scale points.

In fact, in the case of *most*, even ordinal level measurement is not required. The degrees on an ordinal scale are totally ordered relative to one another, in that for any two distinct degrees *a* and *b*, either $a > b$ or $b > a$ obtains. Formally, such a structure is a (strict) total order. More weakly ordered structures are also possible. As a particular case, consider a scale in which the degrees are conceptualized not as points but as ranges or Gaussian distributions, and where the 'greater than' operation $>$ is a function of amount of (non-)overlap between degrees, such that $a > b$ holds iff the range encompassed by a entirely exceeds the range encompassed by b. Such a structure is shown schematically in Figure 2c. Intuitively, we might think of this as a scale where $>$ encodes a 'significantly greater than' relationship. Formally, a scale of the sort described here corresponds to a semi-order on individuals (Luce 1956; van Rooij 2011), an ordering relationship in which the 'greater than' relationship is transitive, but the 'indifference' relationship is not.[9] I will therefore refer to such a structure as a semi-ordered scale.

An underlying scale with this type of semi-ordered structure would be sufficient to support the logical form for *most* in (21a), which would be evaluated as true in the case where the measure assigned to $A \cap B$ exceeded by some adequate margin the measure assigned to $A - B$ (the size of the margin depending on the spread of the Gaussian). But that is precisely the situation in which we have seen that *most* is used, namely in the case where the proportion of As that are B is substantially greater than 50%.

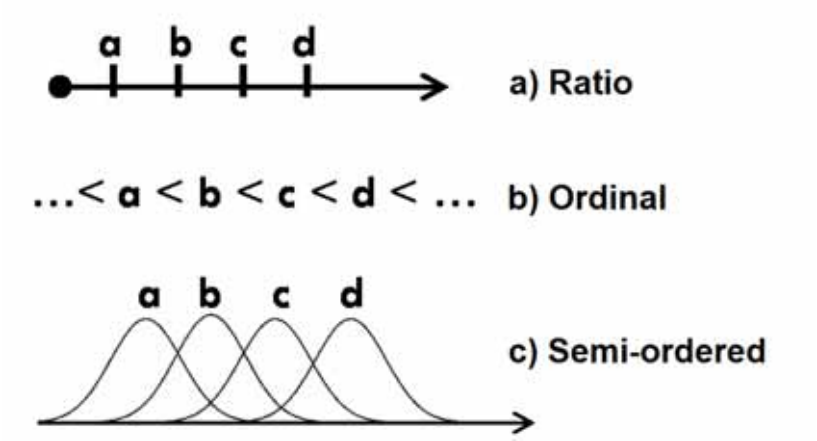
That the meaning of numerals can be related to Gaussian distributions rather than points on a scale is suggested by Krifka (2009), who invokes this idea in accounting for the approximate interpretation of round numbers (e.g. the use of *100* to mean 'about 100'). Thus there is independent reason to think that a structure of

[9] In formal terms, a structure $\langle S, \succ \rangle$ is a **semi-order** iff $\forall x, y, z, v, w \in S$:

(1) a. $\neg(x \succ x)$
b. $((x \succ y) \wedge (v \succ w)) \rightarrow ((x \succ w) \vee (v \succ y))$
c. $((x \succ y) \wedge (y \succ z)) \rightarrow ((x \succ v) \vee (v \succ z))$

See van Rooij (2011) for further details.

Figure 2: Alternate scale structures



the form described above may play a role in the interpretation of natural language quantity expressions. But more fundamentally, such a semi-ordered scale has a close parallel in humans' mental representation of quantities. I turn to this now.

3.3 A parallel from the psychology of number cognition

A large body of research supports the existence of two separate cognitive systems for the representation and manipulation of quantity (see especially Dehaene 1997; Gallistel and Gelman 2000; Feigenson et al. 2004).

The first, which could be termed the 'precise number system', allows the representation of exact number, and is involved in calculations such as addition, subtraction, multiplication and division using symbolic notation. When you solve a mathematical problem such as $327 \times 809 = ?$, for example, you are using this precise number system. The representation and processing of exact number is closely linked to linguistic ability; these capabilities are present primarily in verbal humans, and may be lost in patients with aphasia.

In addition to this capacity to represent precise quantity, however, humans also possess a second system which has been termed the 'approximate number system' (ANS). Under this system, (approximate) quantities are thought to be encoded as patterns of activation on the equivalent of a 'mental number line'. These essentially analog representations can form the basis for approximate calculations such as addition, and – importantly for the present topic – for the comparison of

quantities.

The ANS is developmentally and evolutionarily more basic than the ability to represent precise numerosity. It is operational not just in adults who possess a system of number words, but also preverbal infants, who have been shown to exhibit approximate numerical abilities (Xu and Spelke 2000). Likewise, members of societies without complex number systems have been found to perform successfully on tasks involving approximate quantity comparison and arithmetic, but not those involving precise operations (Pica et al. 2004). The approximate system may be preserved in aphasiacs who have lost the ability to perform exact mathematical operations (Dehaene and Cohen 1991). Non-human animals, too, show similar approximate numerical abilities (Dehaene et al. 1998). Furthermore, in verbal/numerate adults, the role of the approximate number system is not superseded by the possession of learned, precise numerical abilities; for example, when verbal/numerate subjects are presented with tasks that disallow precise counting, they nevertheless are able to judge and compare quantities approximately (Buckey and Gilman 1974).

The hallmark of the ANS is that its operation is characterized by distance and size effects effects. When two quantities are compared in a task where time constraints preclude preclude counting, subjects' response times decrease and their accuracy improves as the numerical distance between the two quantities increases (distance effect). Similarly, for a given numerical distance between two quantities, response times decrease and accuracy improves as the magnitude of the numbers themselves decreases (size effect). Both of these effects are accounted for by the Weber-Fechner law, which in simple terms states that the differentiability of two stimuli is a function of the ratio of their measures. This pattern of ratio dependence has been captured via a model in which representations generated by the ANS are modeled by Gaussian curves with either linearly increasing center values and linearly increasing standard deviations, or logarithmically increasing center values and constant standard deviations (Feigenson et al. 2004). In either case, the differentiability of two values is a function of the degree of overlap of their curves. The representations corresponding to two pairs of values with comparable ratios (e.g. 16 vs. 8 and 12 vs. 6) will have equal overlap, and thus these values will be equally differentiable. Two numerosities whose curves largely overlap are not differentiable via the ANS, or perhaps more accurately, only differentiable in a noisy and stochastic way.

It should be readily apparent that the characteristics of this widely accepted model of humans' approximate numerical abilities are a close match to the semi-ordered scale structure posited to potentially underlie the verification of the truth

conditions of *most*. That is, the purely formal structure which yields an intuitively correct interpretation for *most* has a direct counterpart in the mental representation of number.

The idea that there is a link between the interpretation of *most* and the ANS is not a new one. Pietroski et al. (2009) make this connection explicitly. Through a series of experiments, they demonstrate that verification of a sentence of the form *Most of the dots are yellow* exhibits size and distance effects that can be characterized by the Weber-Fechner law. Thus there is evidence that the ANS may be invoked in at least in the processing of *most* sentences. The connection is made tighter by Halberda et al. (2008), who show that among young children, understanding of *most* is independent of knowledge of large numbers. Specifically, children who do not know the meaning of number words larger than *two* may still be able to evaluate a sentence of the form *Most of the crayons are yellow*, but only if the ratio between the yellow and non-yellow crayons is sufficiently large, suggesting that the children are drawing on their approximate numerical capabilities. This latter study provides particularly nice evidence that the semantics of *most* are not directly based on the representation of precise set cardinality. From a different perspective, Fults (2011) proposes that scales whose structure models the output of the ANS play a role in adjectival semantics (see below for further discussion). In what follows I will argue further that such a scale plays is involved in the interpretation of *most* as well.

3.4 Summary

To summarize, the central idea developed in this section is that *most* and *more than half* have distinct logical forms that, while superficially equivalent in their truth conditions, nonetheless place different requirements on the structure of the underlying scales that support their assessment. Three types of scales have been discussed – ratio, ordinal, and semi-ordered – which differ in the nature of the ordering relationship in place, and therefore in the types of comparisons they support. These three scale types are represented visually in Figure 2. The claim is that the logical form for *more than half* requires the first of these possible scale structures (ratio), while that for *most* allows the second or third (ordinal or semi-ordered). In the next two sections, I discuss how this simple distinction is able to account for the very different behavior of the two quantifiers.

4 *More than half*

I begin with *more than half*, which turns out to be the more straightforward of the two cases. A central idea developed above was that the logical form of *more than half* assumes measurement at the ratio level. The felicitous use of a sentence of the form *'more than half of A are B'* thus requires that we can, at least in principle, map the sets (or portions of matter) $A \cap B$ and A to points on a ratio scale. If we are in the domain of count nouns, and the dimension in question is cardinality, then the relevant scale is the natural numbers, and the mapping function boils down to simple counting of set members. If we are dealing instead with a mass noun and thus a dimension other than cardinality (e.g. weight, area, energy use, etc.), the mapping will correspond to some other sort of ratio-level measurement. In what follows, I show that all of the patterns summarized in Section 2.3 can be accounted by this requirement.

Distribution. As discussed above, the distribution of *more than half* is restricted in several ways relative to that of *most*. I propose that all of these various restrictions derive from a common source, namely the basic requirement for ratio level measurement. Let us begin with the syntactic and semantic restrictions on the first argument of *more than half* (i.e. *A* in the above formalism). What separates the felicitous and infelicitous cases is whether *A* can be measured in such a way to produce a value that can be meaningfully divided by two. Recall that the good examples of *more than half* involved definite descriptions (e.g. *more than half of the men detained at Guantanamo*), plurals with a group reading (e.g. *more than half of teens surveyed*), and mass nouns involving dimensions which have an established unit of measure (e.g. *more than half of home energy use*, where the unit of measure might be kilowatt hours.) These are united by two characteristics which together guarantee measurement at the appropriate level is possible. First, they refer to a fixed set of entities or abstract individual, localized in space and time. And second, the structure of that set/individual is such to allow counting or some other form of extensive measurement.

In the case of nominals that are not felicitous as the first argument of *more than half*, one or the other of these two criteria is not met. Consider first count noun pseudopartitives where the nominal has a kind, rather than group, interpretation (e.g. *teens*, *people*). Here it is the first criterion that it is not satisfied. When nominals of this sort occur with a quantifier, (*all teens*, *most people*, etc.), the intention is typically not to make a statement about the totality teens, people, etc. in the actual or any other world. Rather, the domain of quantification is

in some way contextually restricted (Stalnaker 1972; Westerståhl 1984; Fintel 1989). But precisely how it is restricted is underspecified. That is, the context does not provide a single set whose cardinality could be computed. As a result, quantification with *more than half* is not possible.

Conversely, in the case of plural nominals with non-enumerable domains (e.g. ?*more than half of neighborhoods in Port-au-Prince*), the issue is with the second criterion. While the totality of these domains may be fixed, the individuals members are vague, and cannot be individuated in a non-arbitrary way, with the result that the set cannot be fully enumerated. Thus it is not possible – even in principle – to calculate a set cardinality which could serve as input to an operation of division by two.

A similar explanation can be extended to mass nominals where the dimension in question cannot be measured quantitatively (as in ?*more than half of racism/the racism in America today*). Absent a standard unit of measure, it is not possible to map a domain of this sort to a numerical value that can be divided by two.

This last example, however, raises a question as to the nature of the scale structure requirement imposed by *more than half*. Specifically, is the prerequisite ratio level measurement, or is it the availability of an established unit of measure? These two are not the same. Findings from the classic psychophysics literature (Stevens 1975) seem to demonstrate that even phenomena and sensations for which we lack quantitative measures, such as saltiness, roughness, or pain, may nonetheless be measurable at the ratio level (though this conclusion has been challenged; see e.g. Poulman 1989). One might plausibly propose that even if racism and the like are not associated with numerical scales, they too can nonetheless measured at the ratio level. In apparent support of this, expressions of this type can occur with proportional modifiers such as *twice*, which are limited to ratio scales (22a). But a closer look suggests examples such as these involve a loose use of *twice* to mean something like 'a lot more'. As evidence, the corresponding examples with 'more than twice' show the same sort of infelicity as described above for *more than half*; for example, (22b) has the odd effect of suggesting that racism is a unidimensional phenomenon that can be quantitatively measured.

(22) a. There is twice as much racism there as here
b. ??There is more than twice as much racism there as here

So here too we can conclude that the underlying scale lacks the structure to support the true comparison of ratios – the characteristic of a ratio scale.

The previous discussion pertained to the domain of quantification. A similar account can be applied to restrictions on the sentential predicate. The logical form

of *more than half* in (21b) also requires that the number of A that are B can be counted (or in the mass case, that the amount of A that is B can be measured). The consequence is that the predicate B must defined with sufficient precision that it can be determined which individual A do and do not have property B. From this follows the infelicity of vague predicates with *more than half*.

'Survey results' interpretation. In Section 2 it was noted that the use of *more than half* is commonly backed up by some type of supporting data, and correspondingly that sentences with *more than half* tend to have a 'survey results' feel, implying that the results of some sort of survey or analysis are being reported. The present analysis gives a clue to why this is. The semantics of *more than half* are closely tied to quantitative measurement – counting or some other sort of ratio level measurement. It is then not surprising that *more than half* is used in cases where measurement of this sort has in fact been conducted (e.g. via a survey), and conversely that we tend to infer from the speaker's or writer's choice to use *more than half* that such data are available.

Percentage range. Let us consider finally the range of percentages for which we find *more than half* used. A ratio scale supports precise comparison. In particular, assuming the domain *A* is large enough (relative to the unit of measurement), it will be possible to find subparts of that domain whose measure is arbitrarily close to half that of *A*. To relate this to the logical form of *more than half*, it will thus be possible to find cases where $\mu_S(A \cap B)$ is only slightly greater than $\mu_S(A)/2$. It is therefore not surprising to find *more than half* used for proportions only slightly greater than 50%, exactly what was observed in the corpus data. This may seem to be an obvious point, but in the next section we will see that in the case of *most*, which assumes a weaker ordering than a ratio scale, such precision of measurement cannot be guaranteed, with consequences for the proportions for which the quantifier is employed.

But the precise lower bound for *more than half* is only part of the story. Recall that *more than half* is rarely used for proportions above two thirds or so. This cannot be explained on the basis of precision of measurement. Rather, I propose that this pattern derives from competition between *more than half* and other expressions of proportion, an analysis that follows Gricean lines (Grice 1975; Horn 2005). If we are in a situation that supports an utterance of *more than half* – namely one in which the relevant sets or entities are measurable at the ratio level – then there are a range of alternative expressions that, depending on the percentage

to be conveyed, could also have been used. Examples of such alternatives include *a quarter*, *two thirds*, *8-in-10*, and so forth. By standard (neo-)Gricean reasoning, the use of *more than half* will be pragmatically restricted to cases where no stronger alternative is available. In particular, since *more than two thirds* and *more than three quarters* represent equally lexicalized but stronger alternatives to *more than half*, the felicitous use of *more than half* will be limited to cases where the proportion in question is below this, that is, to the range from half to roughly two thirds. Conversely, the hearer can infer from the speaker's choice of *more than half* that no stronger alternative is available, and thus that the percentage falls within this range. The conclusion, then, is that the interpretation of *more than half* receives an upper bound via scalar implicature.[10]

One might argue that there is an alternate and simpler explanation for these facts, namely that the interpretation of *more than half* is pragmatically upper bounded via competition not with numerical expressions such as *two thirds*, but rather with *most*. This is made initially plausible by the fact that the approximate upper bound for the use of *more than half* corresponds quite closely to the approximate lower bound for the use of *most*. Evidence against this possibility is provided by looking at the corresponding data for *less than half*. There is no counterpart to *most* that *less than half* could compete with. But nonetheless, *less than half* is almost exclusively used for percentages quite close to half (in 98% of corpus tokens the percentage is in the range 35-49%). This can be explained via competition between *less than half* and lower alternatives such as *one third*; for example, if I know the percentage in question to be in the high twenties, then it would be more informative to assert *less than a third* rather than *less than half*. The more parsimonious explanation is then that a similar dynamic is in play in the case of *more than half*, whose range of application is restricted by competition with higher numerical expressions of proportion (and not with *most*).

Summary. The corpus analysis identified a number of restrictions on the felicity of *more than half*, relating to both the syntax and the semantics of the nominal expression providing the domain of quantification, as well as the semantics of the sentential predicate. While these may seem to be unrelated patterns, we have seen here that they all can be traced back to the assumption of measurement at the ratio level, which derives directly from the quantifier's logical form. The same

[10] It has been claimed that scalar implicatures do not arise with modified numerical expressions. However, Cummins et al. (submitted) show that once scale granularity is taken into consideration, such implicatures are in fact observed for expressions such as *more than 100*.

factor, when combined with a Gricean story of competition among alternatives ordered by informativeness, also accounts for the range of proportions for which *more than half* is used. That is, all of the patterns summarized in Section 2.3 lend themselves to a measurement theoretic explanation. In the next section, we will see why the picture is quite different in the case of *most*.

5 *Most*

While the logical form of *more than half* presupposes measurement at the ratio level, that for *most* assumes only a qualitative ordering on individuals with respect to the relevant dimension, one that could be represented via an ordinal or semi-ordered scale. To draw on the analogy introduced at the start of the paper, such an ordering can be likened to the outcome of comparing pairs of objects on a balance scale to determine which weighs more, without assigning numerical measures to any object individually. Again, the patterns observed in the corpus analysis follow from this.

Distribution. Since the scale structure required for *most* is more weakly ordered than that required for *more than half*, it is to be expected that the distribution of *most* is broader, and this is precisely what is observed in the corpus data. The claim, then, is that the contexts that allow *most* but not *more than half* are precisely those where the level of measurement is weaker than ratio level. But this line of thinking rests on the fairly unconventional idea that number and other dimensions of amount can be measured at something less than the ratio level. In what follows, I argue that this is in fact the case, and that it is what is involved in the relevant uses of *most*.

Consider again the sorts of nominal expressions which occur with *most* but not *more than half*. This includes plurals with a kind interpretation (e.g. *teens*), as well as abstract nouns whose denotations are not enumerable or otherwise quantitatively measurable (e.g. *neighborhoods, racism*). As discussed in the preceding section, something about these domains precludes extensive measurement. But in spite of this, their subsets can be compared in magnitude to one another, as is seen in examples such as the following:

(23) a. More teens have cell phones than laptops.
b. And there are more neighborhoods in Chicago than in SF, did you explore those? (http://www.city-data.com/forum/city-vs-city/819221-

shopping-chicago-vs-san-francisco-5.html)

c. There is more racism in Ohio than in Alabama, which some may not believe. (http://blog.cleveland.com/metro/2007/10/cleveland_schools_closed_thurs.html)

Similarly, the example below shows that despite the vagueness of the gradable adjective *wealthy* – which prevents its occurrence as a sentential predicate with *more than half* – the wealthy subset of a given set can be compared in magnitude to other subsets.

(24) Last year, there were more wealthy visitors to the park than poor ones

In that it is possible to make comparisons of this sort, an ordering can be induced on a domain, one that could be represented by a function that maps entities within that domain to points on some scale S. And this is sufficient to support the logical form of *most*, which requires only a (perhaps partially) ordered set of degrees as the range of the measure function μ_S.

It seems reasonable to say that how we make comparisons of the form in (23) and (24) does not fall within the domain of semantics proper, but rather involves the interface with other conceptual systems. In his foundational work on degree semantics, Cresswell (1977, p. 281) argues "[i]t is not, in my opinion, the business of logic or linguistics (at least syntax) to explain how it is that we make the comparisons that we do make or what the principles are by which we make them." But putting aside the psychological processes on the basis of which speakers make assertions of this sort, and focusing on what the world must be like to establish the truth or falsity of each, we can derive insights into the nature of the ordering relationships that they give rise to, that is, the structure of S.

A case can be made that all of these sorts of comparisons involve generalization over specific instances. Each of the classes of expressions discussed here is characterized by some sort of indeterminacy or underspecification in interpretation. This is perhaps most apparent for vague predicates such as *wealthy*, where what is underspecified is the standard of comparison or threshold (how much money one must have to count as wealthy), but it extends to the other examples as well. In the case of a plural nominal with a non-enumerable denotation, what is underspecified is how the members of the domain should be individuated, for example, how we choose to partition a city into neighborhoods. For the case of a plural kind nominal, it is which members of the kind are under discussion that is not fully specified; in an example such as *Most teens want to fit in with their peers*, the question is which teens we are talking about. Such indeterminacy might

be represented as follows, where Π is a partition of an entity into non-overlapping parts (Schwarzschild 1996), *DOM* is a domain variable (Fintel 1989), and *c* is a variable ranging over contexts that represent different ways of specifying the relevant element.

(25) a. $\lambda x.WEALTH(x) \geq d_c$
b. $\Pi_{neighborhoods,c}(Chicago)$
c. $[\![teens]\!] \cap DOM_c$

But despite these various sorts of indeterminacy or underspecification, comparisons can be made by generalizing across the ways that the indeterminacy can be resolved, that is, across different choices of c in (25). In uttering (23a), a speaker might have only a general sort of teens in mind, rather than a specific set; but the sentence would in any case be true if for any way of choosing a specific set that was consistent with the speaker's intention, the mobile-phone-owning subset outnumbered the laptop-owning one. For the truth of (23b), it is likewise sufficient that whichever plausible way we choose to partition Chicago and San Francisco into neighborhoods, the latter outnumbers the former. And (24) would be judged true if, no matter which specific thresholds we set for *wealthy* and *poor*, a choice which is constrained but not totally determined by world knowledge, the wealthy group of visitors outnumbers the poor group.

Put differently, sometimes indeterminacy of the sort discussed here simply does not matter, because a particular relation holds no matter how we choose to make the imprecise precise. When this is the case, an ordering can be established between two subsets of a larger domain, without needing to first resolve the indeterminacy. But importantly, an ordering of this nature is essentially qualitative. Members of a particular set of teens can be counted, as can elements of a particular partitioning of a city into neighborhoods. But in generalizing across the particulars, only a 'greater than' relationship is established. That is, comparison of this sort is akin to the analog process of comparing the weights of two objects on a balance scale to determine whether one or the other is heavier. In turn, if an ordering established this way were represented numerically, the resulting scale would be at most ordinal in level; and it is not even clear that it would be totally ordered.

A similar point can be made with regards to mass nouns such as *racism*. Here, what is at issue is the inherently multifaceted nature of the concept: racism is not a unidimensional phenomenon, but is rather made up of a cluster of attitudes and behaviors, the totality of which one would be hard pressed to specify. But the

truth of an example such as (23c) can nonetheless be evaluated, by generalizing across these various attitudes and behaviors and determining whether in light of all of them Ohio places ahead of Alabama. In this case, it can be shown that the ordering that results need not be total. Suppose as an (over-)simplification that the degree of racism of a city or region is a function of just two factors, and suppose further that one city/region ranks above another in racism if and only if its measures on both dimensions are higher. Now consider the following situation: City A has a measure of 20 on the first dimension and 10 on the second (assuming some appropriate unit of measure for each); City B has measures of 10 and 20; and City C has measures of 25 and 15. Then City A ranks neither higher nor lower than City B, and city B likewise ranks neither higher nor lower than city C. But City C ranks above City A. That is, transitivity of indifference fails.

This latter example recalls discussions of multidimensional adjectives such as *clever*, whose applicability is a function of multiple criteria rather than a single unitary dimension (for example, cleverness might involve ability with numbers, quick wittedness, ability to manipulate people, and so forth). As discussed by Kamp (1975), Klein (1980) and van Rooij (2010), if one individual is to be considered more clever than another overall only if he/she has a higher rank on each of the individual dimensions, then the resulting ordering is only a partial order.

The picture that emerges is this: entities that due to some inherent indeterminacy cannot be extensively measured – for example the totality of racism in Ohio or Alabama, the totality of neighborhoods in San Francisco, or the totalities of teens who do and do not want to fit in with their peers – can nonetheless be compared with one another with respect to magnitude. I have suggested here that such comparison proceeds via a process of generalization over ways the indeterminacy can be resolved, producing an ordering on a domain which could be represented by a scale that is at the ordinal level, or only partially ordered. Such a scale is not sufficient to support the evaluation of the logical form for *more than half* (which requires ratio level measurement). But it is sufficient for the evaluation of the logical form of *most*, which for truth requires merely that the measure assigned to $A \cap B$ be ordered above that assigned to $A - B$.

Generic interpretation. In Section 2 it was pointed out that *most* in combination with a kind-denoting plural nominal has a generic feel. The preceding discussion sheds light on this. Generics do not report specific facts, but instead regularities that summarize groups of facts (Krifka et al. 1995). In this sense, *most* sentences <u>are</u> generic. Whereas a sentence such as *Teens want to fit in with their*

peers expresses a generalization over individual teens, the equivalent sentence with *most* (*Most teens want to fit in with their peers*) expresses a generalization over the ways of making the denotation of *teens* precise enough for counting. The prediction from this is that the *most* sentences – while generic in nature – are not truth-conditionally equivalent to their counterparts with bare plurals. That this is the case is of course well known (Carlson 1977). For example, the truth of *Mosquitoes carry the paramecium that causes yellow fever* does not require that more than 50% of the relevant creatures be disease carriers (fortunately, only a fraction of them do); conversely, *Crocodiles die before they attain an age of two weeks* is clearly false, despite the fact that the majority (i.e. most) crocodiles do in fact meet this sad fate.

Here, a question arises as to whether and how the posited generalization process should be be semantically represented. As described above, comparison based on generalization over particular sets or measures serves as the basis for constructing the scale *S* with reference to which a *most* sentence can be evaluated. An alternate possibility would be to build quantification over particular sets/measures directly into the logical form. For example, suppose C is a set of contexts that represent different acceptable ways of making *wealthy* precise. Then (26a) might be taken to have the logical form in (26b) (putting aside how this might be compositionally derived).

(26) a. Most of the visitors to the park were wealthy

b. $\forall c \in C[\mu_{\#}(\{x : visitor(x) \wedge WEALTH(x) > d_c\}) > \mu_{\#}(\{x : visitor(x) \wedge \neg WEALTH(x) > d_c\}]$

A similar idea might be implemented within a supervaluationist framework (Fine 1975; Kamp 1975), where the (super-)truth of a sentence of the form in (26a) in a partial model *M* requires its truth in all completions of *M*. But on either view, we would be forced to explain why a similar account could not be extended also to *more than half*, allowing it to occur in the same range of contexts where we find *most*. The desired distinction is obtained if we take the generalization process to play a role not at the level of logical form or interpretation relative to a model, but rather in the underlying operation of comparing individuals. The scale that is so derived has an ordinal or partially ordered structure, and as such supports the logical form of *most* but not that of *more than half*.

Percentage range. Having discussed how to account for *most*'s broader distribution relative to *more than half*, let us now consider its interpretation in terms

of proportion conveyed. As discussed in Section 2, *most* is typically not used for proportions very close to 50%. I would like to propose that this pattern derives from its interpretation relative to the sort of semi-ordered scale described in Section 3, one in which degrees have the form of Gaussian distributions rather than points, and the greater than relationship $>$ requires non-overlap of distributions. In this case, a sentence of the form *Most A are B* will be evaluated as true iff the degree assigned to $A \cap B$ sufficiently exceeds that assigned to $A - B$ so that the ranges do not overlap. This in turn will be the case iff the proportion of A's that are B is sufficiently greater than 50%, precisely what was seen in the corpus data.

From the preceding discussion it should be clear that if the situation is such that measurement produces a semi-ordered scale, this is sufficient to support the logical form of *most*. And indeed, I have argued that some cases where *most* is used, the points on the corresponding scale are in fact not totally ordered relative to one another. This makes it quite plausible to look to scale structure for an explanation of this aspect of *most*'s interpretation. But of course, *most* is not used only in situations where the underlying scale is not totally ordered. The logical form of *most* in (21a) allows its interpretation relative to a semi-ordered scale, but it does not require this to be the case. Nothing rules out the use of *most* in a situation where measurement is at the ordinal or ratio level. And this is the right result, since as demonstrated in Section 2, *most* is clearly used in cases where counting or other form of ratio-level numerical measurement is available (e.g. in the reporting of survey data). The issue is that even in these situations, the felicitous use of *most* requires that there be a 'significant' difference between the measures of the two sets or entities compared. In fact, it is this very type of example, as in (6), that provided the evidence that *most* is typically interpreted as a proportion considerably greater than half. It seems to be the case that even when precise, ratio-level measurement is available, *most* is used and interpreted as if it were not. That is, the default interpretation of *most* is relative to a semi-ordered scale.

I propose that this pattern arises via pragmatic strengthening. Note first that the strong tendency for the use of *most* to be restricted to situations where there is a significant difference between measures is reminiscent of cases of what Horn (1984) terms R-based implicature, where a more general predicate is pragmatically restricted or narrowed to stereotypical instances. Such implicatures derive from the Horn's R Principle: "say no more than you must". Examples of R-based implicatures discussed by Horn include the strengthening of ability modals (such that *John was able to solve the problem* R-implicates that he in fact solved it), the restriction of lexical causatives such as *kill* to cases of direct causation, and in the

lexical domain, the inference from *John had a drink* that the beverage in question was alcoholic. Krifka (2007) proposes a similar mechanism as responsible for the strengthening of antonym pairs such as *happy/unhappy* from contradictories to contraries. Horn notes also that R-implicatures may sometimes become conventionalized. This occurs in particular in language change, as for instance in the narrowing of the denotation of *deer* (Old English *deor*) from an object of the chase generally to a prototypical exemplar of that class.

Building on this, there is reason to believe that comparison of quantities with respect to a semi-ordered scale structure is in fact the stereotypical case. Recall that the structure of a semi-ordered scale is parallel to that of the most widely accepted model of the output of the Approximate Number System (ANS), humans' most basic cognitive system for the representation and manipulation of quantity information. Approximate comparison is more basic than precise comparison. Our approximate numerical abilities (i.e., the ANS) emerged evolutionarily before the ability to represent precise number, develop earlier in the child, and are activated even in tasks involving precise numerical reasoning. A semi-ordered scale thus models our most primitive representation of quantity.

I therefore suggest that the lexical entry for *most* is as in (21a), according to which it is true in the 'just over 50%' case, but that its evaluation is pragmatically strengthened to one relative to a semi-ordered scale structure that models the output of the ANS, yielding the 'significantly greater than' interpretation discussed above. The observation that some, though not all, speakers find the 'significantly greater than' aspect of *most*'s semantics to be truth-conditional in nature suggests that this strengthening is of the conventionalized variety.

The last piece of the puzzle to address relates to the upper bound for *most*, namely why it can be used for proportions approaching 'all'. Recall that use of *more than half* is typically restricted to proportions quite close to half, an effect that was attributed to competition with higher numerical expressions such as *two thirds*. Apparently the availability of these expressions does not constrain the interpretation of *most* in the same way. The reason, I would argue, is that they too assume measurement at the ratio level, and thus do not compete directly with *most*. That this is the case is supported by examples such as the following, which demonstrate that in contexts where *more than half* is infelicitous (or takes on a survey results interpretation), so too are expressions such as *two thirds*, *6-in-10* and so forth.

(27) a. Most teens want to fit in with their peers
b. ?More than half of teens want to fit in with their peers

c. ?Three quarters of/more than two thirds of/6-in-10 teens want to fit in with their peers

Since *most* does not compete directly with expressions such as these, they cannot provide the basis for pragmatically restricting its interpretation. Instead, the possible competitors for *most* seem to be other quantifiers that express relationships between sets, and that are commonly organized on a Horn scale (Horn 1989, 2005).

(28) a. no . . . some . . . many . . . most . . . (almost all) . . . all
b. No/some/many/(almost) all teens want to fit in with their peers

As seen here, the only higher competitor to *most* is *all*, or perhaps *almost all*. And this seems correct, in that (as seen in Section 2) *most* is used for proportions all the way up to 90+%, at which point *almost all* or even *all* would be felicitous.

Summary. The logical forms I have assumed for *most* and *more than half* are, on the surface, truth conditionally equivalent. This might lead us to expect that the two would be interchangeable, an expectation that is amply refuted by the corpus data. It is only when we introduce the idea of variation in the structure of the underlying measurement scales that a coherent explanation emerges. The difference in the nature of the scales assumed by the two quantifiers accounts for the the broader distribution of *most* relative to *more than half*, while the possibility of evaluating *most* relative to a semi-ordered scale, when coupled with otherwise attested pragmatic principles, accounts for its tendency to be restricted to proportions considerably greater than half.

The scale structure account developed here also sheds new light on Hackl's (2009) findings on the online processing of sentences with *most* and *more than half*. Hackl's experimental methodology, which involved the evaluation of sentences of the form *'Most/more than half of the dots are blue'* against sequentially exposed dot arrays, was designed such that participants could easily establish whether a 'greater than' relationship obtained between two sets of dots, but less easily establish the number of dots of each color. It is therefore not surprising that the *most* sentences, whose logical form can be assessed relative to a simple 'greater than' ordering (ordinal), were verified more quickly that the *more than half* sentences, whose assessment requires determining the cardinality of sets (ratio). There is thus no need to posit a separate verification strategy associated with each quantifier; rather, the type of information required for the verification of each

derives directly from its logical form, such that a task that makes one sort of information more salient than another will favor one quantifier over the other.

Importantly, there is no *prima facie* reason to think there would be a connection between syntactic distribution, interpretation in terms of proportion, and verification time in online sentence processing. That the observed divergences in all of these areas can be explained in terms of scale structure speaks in favor of this type of account.

6 Conclusions and connections

There is no doubt that speakers sometimes talk about degrees, and correspondingly there is no doubt that degrees must somehow be semantically represented. Most current semantic theories that deal with measure, gradability and comparison in some way assume degrees as part of the ontology. But there is little consensus as to the structure of the domain of degrees.

In what might be considered the foundational work on degree semantics, Cresswell (1977) makes only weak assumptions about how degrees are ordered into scales, remarking that "[i]t is tempting to think of $>$ as at least a partial ordering (i.e. a transitive and antisymmetric relation); whether it should be strict or not or total or not seems unimportant, and perhaps we should even be liberal enough not to insist on transitivity and antisymmetry" [p. 266]. But other work within this general framework typically assumes that the degrees on a given scale are totally ordered relative to one another (Bartsch and Vennemann 1973; Fox and Hackl 2006; Kennedy 2007). Fox and Hackl (2006) in particular make the stronger claim that all measurement scales – including the scale of cardinality – are formally dense, having a structure isomorphic to the rational or real numbers. Furthermore, the running examples in many works involve gradable adjectives such as *tall* or *expensive*, which encode dimensions corresponding to familiar ratio level scales with numerical units of measurement (e.g. height in cm; cost in dollars).

At the same time, it is now well recognized that measurement scales vary in their structure, and that this has linguistic effects. Kennedy and McNally (2005) discuss the distinction between open and closed scales, and show that this has consequences for the interpretation of gradable adjectives in their positive forms, as well as for the distribution of degree modifiers. For example, *completely* occurs only with adjectives that map their arguments to scales that are closed on their upper end (*completely flat* vs. ?*completely bumpy*, ?*completely tall*), while *very*

occurs most felicitously with open scale adjectives (e.g. *very tall*). From another perspective, Bale (2008) argues that the presence or absence of numerical degrees determines the interpretations available for interadjective comparisons such as *the table is longer than it is wide* and *Sue is smarter than Hilda is beautiful*.

The present work points to another dimension along which measurement scales vary, namely the strength of the ordering relationship $>$ on the degrees that constitute them. It is not sufficient to posit that scales are totally ordered sets of points. Some have more structure than this, specifically a fixed unit of measurement and perhaps a zero point, and thus correspond to interval or ratio level measurement. And some scales have less structure than this, in that degrees are not totally ordered relative to one another (a possibility Cresswell left open). The comparison of *most* and *more than half* shows that this aspect of scale structure also has linguistic consequences. *More than half* (and by extension other numerical expressions of proportion) requires a ratio level scale, and is thus restricted to use in cases where entities can be mapped to a scale of this sort. *Most*, by contrast, can be interpreted with respect to a simple qualitative ordering of entities, one which would be reflected by an ordinal or semi-ordered scale. This distinction influences both the distribution of the two quantifiers as well as the interpretations they receive.

There is a further conclusion that arises out of the present analysis. Specifically, the dimension of measurement does not fully specify the scale. On one level, this is of course a trivial observation. Height can be measured in feet or meters, temperature in degrees Centigrade or Fahrenheit, and so forth. In the present study, however, I have argued for a more fundamental sort of variation: the same dimension can be tracked by scales representing different levels of measurement. Such a possibility is not without precedent in the physical sciences. Mineral hardness, for example, can be measured at the ratio level via a sclerometer, or at the ordinal level via an '*x* scratches *y*' procedure. The two weighing procedures discussed at the start of the paper likewise represent two levels in the measurement of weight. Here, we have seen something similar with cardinality or number. We typically think of measuring the size of a finite set of entities by counting its members, that is, by mapping them to the natural numbers. This is a prototypical example of ratio level measurement. But it is also possible to compare set sizes in an analog or qualitative way, based on a simple 'larger than' judgment. The scale that results is ordinal or, if the 'larger than' judgment is insensitive to small differences in set sizes, semi-ordered. We have seen that in some cases only the weaker qualitative level of measurement is possible, but even when actual counting is (at least in principle) possible, we might make judgments in a qualitative way; in

fact, this seems in some cases to be the default option. An analogy introduced by Bartsch and Vennemann (1973) already alludes to this distinction: the number of votes for or against a proposition may be assessed by precise counting, or alternately by listening to the 'ayes' and 'nays'. The first procedure represents ratio level measurement; the second, in that it is insensitive to small differences in number of votes, would in the present terms give rise to a semi-ordered scale. More importantly, these dual possibilities very closely reflect what we know about how humans actually represent number – either precisely and digitally via our linguistically oriented precise number abilities, or as analog magnitudes via the ANS. But the linguistic consequences of this duality have not, to my knowledge, been previously recognized.

If it is the case that scale structures – even those tracking the same dimension – can vary in this way, we should expect to see it reflected in other domains of natural language as well. While it beyond the scope of the present paper to explore this question more generally, by way of closing I will briefly discuss another construction that lends itself to an analysis along similar lines. The topic has to do with the contrast between explicit comparatives such as (29a) and implicit comparatives such as (29b), which has been discussed in particular by Kennedy (2007):

(29) a. Fred is taller than Barney
b. Fred is tall compared to Barney

While (29a) can be felicitously used to describe a situation where Fred's height is just slightly greater than Barney's (say, 1 cm), the felicitous use of (29b) requires there to be a significant difference in the two individuals' heights. This corresponds closely to the distinction between *more than half* and *most*; as discussed above, (30a) is felicitous if Fred has read just slightly more than 50% of the relevant books, but (30a) requires that the number be considerably greater than that threshold.

(30) a. Fred read more than half of the books
b. Fred read most of the books

The contrast in (29) can be accounted for in a parallel way to that in (30) if we take (29a) to involve a ratio scale of height, but (29b) to involve a semi-ordered scale, one where the 'taller than' relationship requires a significant difference in heights. In fact, Fults (2011) argues for very much this analysis, proposing in particular that implicit comparatives involve an 'analog magnitude scale' parallel

in structure to the output of the ANS. van Rooij (2011) similarly proposes that the difference between (29a) and (29b) corresponds to that between strict weak orders and semi-orders, which in turn derives from differences in how predicates behave across comparison classes.

That factors relating to the type of ordering relationship invoked have explanatory value in this domain as well suggests the broader relevance of the type of account developed here.

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