1. Introduction

1.1 The Data

A well-known pattern: *many* and *few* occur both in bare ‘positive’ form (1) and in combination with a range of degree modifiers (2) – the same modifiers that occur with gradable adjectives (3):

(1) a. Many students attended the lecture
    b. Few students attended the lecture

(2) a. Very many/very few students attended the lecture
    b. Professor Jones was worried that too many/too few students would attend the lecture
    c. The speaker was astonished that so many/so few students attended the lecture
    d. More than 100/fewer than 100 students attended the lecture

(3) a. very tall
    b. too tall
    c. so tall
    d. taller than 6 feet


Some little-studied patterns:

(4) a. Frank’s good qualities are many/few
    b. The many/few archeologists I know
    c. A few students (vs. *a many students)
    d. A long few years
    e. Every few years
    f. Professor Jones’ many and important/few and trivial contributions ....

Not accounted for by existing theories

Focus of today’s talk: the ‘differential’ uses of *many/few*, where they occur as modifiers in comparatives:

(5) a. Many more: There were 100 seats in the lecture hall, but unfortunately many more than 100 students showed up for the lecture
    b. Few more: The lecture hall has 500 seats, but few more than 100 students attended the lecture
    c. Many fewer: The whole class of 100 was supposed to attend the lecture, but many fewer than 100 students actually came
    d. Few fewer: ??

(6) a. Many more: Most homes have many more than 3 televisions
    b. Few more: Few more than 400 Sumatran tigers survive in the wild
    c. Many fewer: The latest attempt to count the number of transient vacation rentals on Maui finds many fewer than previously estimated

This sort of recursive degree modification discussed in the earlier syntax literature (Bresnan 1973; Jackendoff 1977), but the facts in (5) - (6) have received little attention in the semantics literature (though see Schwarzchild 2006)

1.2 Goals of Today’s Talk

Propose a semantic analysis of *many/few* as gradable predicates of scalar intervals, which allows a compositional analysis of the complex quantifiers in (5) as well as the simpler constructions in (1)-(2)

Discuss some possible extensions to other data

Explore consequences for degree modification in the quantificational vs. adjectival domains

2. The Analysis

2.1 What Doesn’t Work

The differential constructions exemplified in (5) and (6) are problematic for…

… an analysis of *many/few* as the ‘quantifying determiners’ of Generalized Quantifier Theory -- (type (εt, (εt, ε))) - Barwise & Cooper 1981) or their gradable counterparts (type (d, (εt, (εt, ε))) - Hackl 2000; Takahashi 2006)

In (5) - (6), what 2 sets could serve as the arguments for a quantificational *many/few*?
... an analysis of *many* and *few* as (gradable) cardinality predicates or noun modifiers (type \(\langle \text{et} \rangle\), \(\langle \text{d,et} \rangle\) or \(\langle \text{d,} \langle \text{et,et} \rangle \rangle\) - Milsark 1977; Partee 1989; McNally 1998; Kennedy & McNally 2005; Hackl to appear)

\[
(5c) \text{Many fewer than 100 students attended the lecture}
\]

\[
\text{‘there was a group X composed of 100 students and a separate group Y composed of many students …’}
\]

cf. ‘many fewer than 400 Sumatran tigers survive today’

### 2.2 The Proposal - Intuitively

- In (5), *many* and *few* express properties of an interval on the scale of natural numbers – the interval between the point 100 and that corresponding to the number of students who attended the lecture (#-Students-AtL)

\[
(7)\ a. \text{Many more than 100 students attended the lecture}
\]

\[
\begin{array}{c}
\text{0 1 2 3 4 \ldots n} \\
\text{large} \\
\text{100} \\
\text{#-Students-AtL}
\end{array}
\]

b. Few more than 100 students attended the lecture

\[
\begin{array}{c}
\begin{array}{c}
\text{0 1 2 3 4 \ldots n} \\
\text{small} \\
\text{100} \\
\text{#-Students-AtL}
\end{array}
\end{array}
\]

c. Many fewer than 100 students attended the lecture

\[
\begin{array}{c}
\begin{array}{c}
\text{0 1 2 3 4 \ldots n} \\
\text{large} \\
\text{100} \\
\text{#-Students-AtL}
\end{array}
\end{array}
\]

- The account can be extended to the simpler constructions in (1)-(2) by considering an interval beginning at 0:

\[
(8)\ #\text{-Students-AtL}
\]

\[
\begin{array}{c}
\text{1 2 3 4 \ldots n} \\
\text{many students: large} \\
\text{few students: small} \\
\text{very many/few students: very large/small} \\
\text{too many/few students: too large/small etc.}
\end{array}
\]

Central Claim: *Many* and *few* denote gradable predicates of scalar intervals (cf. Schwarzschild 2006, Rett 2006 for similar proposals based on different data).

### Advantages:

- Parallel treatment of *many/few* and *much/little* as mass quantifiers (9a,b) and modifiers in adjectival comparatives (9c) – the latter of which long been analyzed as degree constructions (e.g. Klein 1982)

\[
(9)\ a. \text{much/little wine} \\
b. \text{much/little more than a gallon of wine} \\
c. \text{much/little taller than Fred}
\]

- Separation *many/few* from predication or quantification over individuals provides path to account for broader range of data

### 2.3 The Proposal -- Formally

#### Preliminaries:

- I assume an ontology that includes degrees as a primitive type (type d)

#### Degrees

\(d\) are points on a scale \(S\) associated with some dimension \(\text{DIM}\), ordered by the ‘greater than’ relationship (>)

- In the case of *many/few*, the relevant dimension is ‘cardinality’, and the resulting scale is that of the natural numbers

- I use the term *interval* to denote a continuous set of degrees.  Formally:

\[
(10)\ \text{A set of degrees } I \subseteq D(d) \text{ is an interval iff } \\
\forall d, d', d'' \in D(d) \text{ such that } d > d'' > d', (d \in I \land d' \in I) \Rightarrow d'' \in I
\]
I assume the framework of Heim (2000, 2006) in which degree modifiers (very, too, -er than n, etc.) are analyzed as generalized quantifiers over degrees (type \( \langle dt, t \rangle \)).

- Provisionally: The positive forms (bare many/few) involve null degree quantifier POS with the following definition (Heim 2006; von Stechow 2006)

\[
\| \text{POS} \| = \lambda d_1. \forall d \in N_S \quad d \in I
\]

Here \( N_S \) is the ‘neutral range’ on the scale \( S \) – the range of degrees that would be considered neither large nor small w.r.t the given context (see also Cresswell 1977; von Stechow 1984; Kennedy 2007) for other views of POS.

**Formal Proposal:**

1) *Many* and *few* denote gradable predicates of scalar intervals (type \( \langle d, \langle dt, t \rangle \rangle \)), with *few* defined relative to the join complementary interval of the original interval (cf. Kennedy 2001; Heim 2006):

\[
\begin{align*}
\| \text{many} \| &= \lambda d_1. \lambda I_1. \quad d \in I_1 \\
\| \text{few} \| &= \lambda d_1. \lambda I_1. \quad d \in \text{INV}(I) 
\end{align*}
\]

where for \( I = [0,d] \), \( \text{INV}(I) = [d, \infty] \)

2) Quantification over individuals arises via function of existential closure (\( \exists \)) associated with a node high within the DP projection (cf. Krifka 1999)

\[
\| \exists \| = \lambda P_1. \lambda Q_1. \exists x. [P(x) \land Q(x)]
\]

3) *Many* and *few* (predicates of scalar intervals) are linked to common nouns (predicates of individuals) via covert measure function COUNT (cf. Schwarzschild’s 2006 Mon; Rett’s 2006 COUNT; Kayne’s 2005 null noun NUMBER)

\[
\| \text{COUNT} \| = \lambda d. \lambda X. \text{COUNT}(X) \geq d
\]

- The role of COUNT is to introduce a degree argument and link it to an individual argument

- Being of type \( \langle d, et \rangle \), COUNT may combine with a common noun (type \( \langle et \rangle \)) via variable identification (Kratzer 1996)

\[
\begin{align*}
\| \text{students} \| &= \lambda X. \ast \text{student}(X) \\
\| \text{COUNT students} \| &= \lambda d. \lambda X. \ast \text{student}(X) \land \text{COUNT}(X) \geq d
\end{align*}
\]

The resulting structure has the following interpretation (give first formally, then more simply):

\[
\begin{align*}
\| 1a \| : & \text{Many students attended the lecture} \\
& \forall d \in N_S \quad \exists X. \ast \text{student}(X) \land \text{AtL}(X) \land \text{COUNT}(X) \geq d \\
& N_S \subseteq [0, \#-\text{Students-AtL}] \\
& \text{‘the number of students attending the lecture exceeds the neutral range’}
\end{align*}
\]

\[
\begin{align*}
\| 1b \| : & \text{Few students attended the lecture} \\
& \forall d \in N_S \quad \exists X. \ast \text{student}(X) \land \text{AtL}(X) \land \text{COUNT}(X) < d \\
& N_S \subseteq [\#-\text{Students-AtL}, \infty] \\
& \text{‘the number of students attending the lecture falls short of the neutral range’}
\end{align*}
\]
Advantages of degree-quantifier analysis of \textit{few} (not fully captured by other accounts):

- Existential quantification without ‘van Bentham’s problem’ (van Bentham 1986)
- Scope splitting

(18) They need few reasons to fire you

‘it is not the case that they need a large number of reasons…’

The Comparative:

 Comparatives must provide intervals to serve as the arguments of \textit{many/few}. For example, for (5a) – repeated as (19a) -- we require the interval in (19b) or, equivalently, that in (19c):

(19) a. many more than 100 students attended the lecture
b. \([100, \# \text{-Students-AtL}]\)
c. \([0, \# \text{-Students-AtL} – 100]\)

 Can be achieved by defining comparative to include an additional degree argument that can be saturated or bound by a degree expression (von Stechow 1984; Kennedy 2001; Schwarzschild & Wilkinson 2002):

\begin{equation}
\| \text{-er than } d \| = \lambda d \lambda I_{d_0} [d \in I \land 0 \leq d' \leq \text{MAX}_{d_0}(I) - d]
\end{equation}

where \(|n| = \text{absolute value of } n\),

\text{MAX}_{\text{dir}} is a maximality operator sensitive to interval direction

Yields (21) - (23) as the logical forms and resulting semantic interpretations for (5a-c):

(21) Many more than 100 students attended the lecture

\([\text{POS}_4 [t_4\text{-many}_3 [(t_3\text{-er than 100})_2 [t_2\text{-many}_1 [t_1\text{-COUNT students attended}]\]]]]\)

100 \in \[0, \# \text{-Student-AtL} \land N_S \subseteq [0, \# \text{-Student-AtL} – 100]\]

(22) Few more than 100 students attended the lecture

\([\text{POS}_4 [t_4\text{-few}_3 [(t_3\text{-er than 100})_2 [t_2\text{-many}_1 [t_1\text{-COUNT students attended}]\]]]]\)

100 \in \[0, \# \text{-Student-AtL} \land N_S \subseteq [\# \text{-Student-AtL – 100}, \infty]\]

(23) Many fewer than 100 students attended the lecture

\([\text{POS}_4 [t_4\text{-many}_3 [(t_3\text{-er than 100})_2 [t_2\text{-few}_1 [t_1\text{-COUNT students attended}]\]]]]\)

100 \in [\# \text{-Student-AtL, } \infty] \land N_S \subseteq [0, 100 – \# \text{-Student-AtL}]}

Computation of logical forms in (21)-(23) involves successive creation and manipulation of scalar intervals.

(24) Many fewer than 100 students attended the lecture (5c)

d\text{-COUNT students attended } \[0, \# \text{-Students-AtL}\]
few \[\# \text{-Students-AtL, } \infty\] by INV in (12b)

\text{-er than 100 } \[0, \# \text{-Student-AtL} – \# \text{-Student-AtL}\] by (20)

many \[0, 100 – \# \text{-Student-AtL}\] by (12a)

POS \text{NS} \subseteq [0, 100 – \# \text{-Student-AtL} – 100]

Ungrammaticality of \textit{*few fewer}: successive application of INV disallowed?!

Unmodified comparatives: I propose the degree argument is existentially bound by an existential operator that picks out a non-zero degree:

(25) More than 100 students attended the lecture

\[[d\text{-er than 100}, [t_1\text{-many}_1 [t_1\text{-COUNT students attended}]]]]

100 \in [0, \# \text{-Student-AtL}] \land \exists d \neq 0. d \in [0, \# \text{-Student-AtL} – 100]

Interim Summary

Present proposal yields correct semantics for complex quantifiers based on \textit{many/few} (5), as well as simpler constructions (e.g. (1), (2))

Below I show it has the potential to account for other facts (as in (4)) – but requires revisiting some assumptions about degree modifiers

3. Extensions

3.1 Predicative \textit{many/few}

\textit{Many/few} predicated of set of degrees associated with DP

(26) John’s friends are many

\[\text{POS}_2 [t_2\text{-many}_1 [t_1\text{-COUNT(john’s friends) } \geq d’]]\]

\forall d \in N_S [\text{COUNT(john’s friends)} \geq d]

\text{NS} \subseteq [0, \# \text{ of John’s friends}]

‘the number of John’s friends exceeds the neutral range’

Note also:

(27) a. Mary considers John tall

"(Hackl 2000)

b. \text{*Mary considers the guests many}

c. Mary considers 20 students many

(R. Fiengo, p.c.)
3.2 (very) few

• In bare form and when modified by some degree modifiers (e.g. very), few may follow a; in combination with other degree modifiers (e.g. comparative; too) it may not.

  (28)  a. A few students attended the lecture
  b. A very few students attended the lecture
c. *A fewer than 100 students attended the lecture
d. *A too few students attended the lecture

• Few vs. a few (see also Solt 2006)

  • A few is not an idiom: other elements can intervene between a and few:

  (29)  a. very few students
  an incredibly few collectors
  a long few days

  • The difference between few and a few is scope related – they can be given paraphrases that differ only in scope of negation (30), and few but not a few shows characteristics of sentential negation, e.g. NPI licensing (31):

  (30)  a. Few students attended the lecture
   ‘it is not the case that there was a large group of students who attended the lecture’
  b. A few students attended the lecture
   ‘there was a not-large group of students who attended the lecture’

  (31)  a. Few students have ever been to one of Prof. Jones’ lectures
  b. *A few students have ever been to one of Prof. Jones’ lectures

• These differences can be accounted for with an analysis under which the negative expression few in a (very) few does not raise to take sentential scope (as few does in (16)), but rather is interpreted in its base position, below the existential quantifier.

  (32)  a. A few students attended the lecture
   $\exists X(\text{student}(X) \land \text{AtL}(X) \land \exists d[\text{POS-few}(d) \land d \in I \land \text{COUNT}(X) \geq d])$
  b. A very few students attended the lecture
   $\exists X(\text{student}(X) \land \text{AtL}(X) \land \exists d[\text{very-few}(d) \land d \in I \land \text{COUNT}(X) \geq d])$

• If the presence of a creates an environment from which few cannot raise, it should block the raising of degree modifiers as well. Furthermore, and crucially, this suggests an explanation for the contrast in (28), on the assumption that –er than 100 and too must raise for interpretability

  ➔ Implies some degree modifiers (POS, very) can be interpreted in situ

4. Implications for Degree Modification in Quantificational vs. Adjectival Domains

• Many/few vs. gradable adjectives – both gradable, but different semantic types:

  (12)  a. $\text{many} = \lambda d I(0!d \in I)$
  b. $\text{few} = \lambda d I(0!d \in \text{INV}(I))$

  (33)  $\text{tall} = \lambda d x_! x \text{is} \text{d-tall}$

• Immaterial if degree modifiers interpreted as generalized quantifiers over degrees

  • Semantics of comparative given in (20) can be extended to adjectival comparatives such as

    (34)  a. very few students
  b. *too few students

  • If degree modifiers can combine locally with gradable expressions (as proposed above for POS, very), raises questions:

    • Gradable expression as argument of degree modifier (e.g. Kennedy 2007)?
      ➔ Would require degree modifier to have different semantic type in case of many/few and gradable adjectives, an unwelcome result

    • Degree modifier as type-d argument of gradable expression?
      ➔ Consistent with referential/quantificational duality in both individual and degree domains

    • Do POS/very (sometimes) have a ‘referential’ type-d interpretation?

  (35)  $\| \text{POS}\| = \text{d}_{\text{standard}}$

   As support, a few has a context-insensitive interpretation: while few is context-dependent, a few is always $\approx 3-4$ (cf. every few days)

5. Conclusions

• Many and few are neither quantificational determiners nor cardinality predicates, but are rather predicates of sets of degrees (intervals)

• This proposal accounts for both complex quantifiers such as few more than 100 as well as simpler constructions, and shows potential to extend to occurrence of many/few in other contexts

• Consequence is that some degree modifiers cannot always be analyzed as generalized quantifiers over degrees
References


