Errors in Pragmatics

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Abstract

In this paper we are going to show that error coping strategies play an essential role in linguistic pragmatics. We study the effect of *noisy* speaker strategies within a framework of signalling games with feedback loop. We distinguish between cases in which errors occur in message selection and cases in which they occur in signal selection. The first type of errors affects the content of an utterance, and the second type its linguistic expression. The general communication model is inspired by the Shannon–Weaver communication model. We test the model by a number of benchmark examples, including examples of relevance implicatures, quantity implicatures, and presupposition accommodation.

1 Introduction

It is an obvious fact that speakers commit mistakes. It is less obvious that this should have any significance for pragmatics. In this paper, we are going to show that the speaker's and hearer's error coping strategies are among the central forces in pragmatics. In particular, we argue that presupposition accommodation and quantity implicatures are products of the speaker and hearer's error handling strategies. However, we will not approach this thesis directly. Instead, we first discuss an interpretation principle, explicitly formulated by Prashant Parikh, but implicitly part of most game theoretic approaches to pragmatics:[1]

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[1] All explanations of pragmatic phenomena proposed by what we may call the Pennsylvania group of game theoretic pragmatics (Parikh, 2001, 2010; Ross, 2006; Clark, 2009)

> "[...] if ρ, ρ' are the shared probabilities of [S]'s intention to convey p, p' respectively, and b, b' are the respective marginal benefits of not conveying p, p' explicitly then it can be shown that p is communicated with certainty if and only if $\rho\, b > \rho'\, b'$." (Parikh, 2006, p. 111)

In the situations we are interested in, the marginal benefit is the differences between the costs of uttering an unambiguous sentence expressing p and a less costly ambiguous one. If the marginal benefits are equal, then Parikh's principle says that p is communicated with certainty by a statement F which is ambiguous between p and p' if, and only if p is more probable than p'. Hence, in the following example, Parikh's principle would predict that the utterance of F unambiguously communicates the same as an utterance of F_A does:

(1) [Doctor's Appointment] Background: John is known to regularly consult two different doctors, physicians A and B. He consults A more often than B. Then S utters one of the following sentences:

- **a)** John has a doctor's appointment at 4pm. He requests you to pick him up afterwards. (F)
- **b)** John has a doctor's appointment at A's practice at 4pm. He requests you to pick him up afterwards. (F_A)
- **c)** John has a doctor's appointment at B's practice at 4pm. He requests you to pick him up afterwards. (F_B)

As F_A and F_B are equally complex, the marginal benefits are identical. Hence, as F_A is more probable than F_B, Parikh's principle predicts that F communicates F_A with certainty. This is obviously not the case. Instead, the natural reaction of the addressee is a clarification request asking for the place where he can meet John.

Common to most game theoretic models is a communication model without feedback loops from addressee to speaker. Communication happens only one way, from the speaker to the hearer who then has to decide about the meaning, or choose an action. Clarification requests are a kind of feedback which allows the addressee to communicate that he did not fully understand the utterance. Clarification requests are not for free. They need a small effort. However, in examples as (1), this additional effort is negligible in comparison to the discomfort caused by miscommunication. We assume that the

are ultimately reducible to applications of this principle. The principle does also hold in other non–evolutionary models: (Benz and van Rooij, 2007; Jäger and Ebert, 2009; Franke, 2009).

clarification request makes the speaker attentive of his mistake, and therefore makes him restate his message in an unambiguous form. We call clarification requests which have these properties, i.e. nominal costs and guaranteed success, *efficient* clarification requests.

The addition of feedback loops in the form of efficient clarification requests will, in itself, not change the validity of Parikh's principle. After all, clarification requests need effort, however minimal it is. If the speaker is known to follow a strategy which involves ambiguous signals, then the addressee's best response to these signals is still to choose the most probable interpretation. It is only after allowing the possibility of errors in the speaker's strategy that efficient clarification requests can take effects. Obviously, that is what addressees do in the Doctor's Appointment example: they assume that the speaker made a mistake.

However convincing the Doctor's Appointment example may be as a counter–example to Parikh's principle, on its own it does not provide enough evidence for establishing an alternative principle, or let alone to prove the contravariant principle saying that addressees always react with clarification requests to ambiguous utterances. In the next section we discuss Parikh's standard example, which seems to support his principle, and a series of examples which seem to be structurally equivalent with this and the Doctor's Appointment example but do not lead to clarification requests. They serve as benchmarks for the theory which we set up in the subsequent sections.

Our aim is a model of communication with noisy speaker strategies, hence, we take as our reference point the Shannon–Weaver model of communication (1949) which we introduce in Section 3. In the same section, the basic concepts associated with the communication model are also introduced, e.g. the notion of *implicature* as well as a version of signalling games with feedback. Section 4 presents the central error models. In the remaining two sections we test our error models with the benchmark examples from Section 2.

2 The benchmark examples

In this section we provide a series of examples which our theory is supposed to explain. We first examine Parikh's main example:

(2) [Parikh (2001, p. 20)] Every ten minutes a man gets mugged in New York. (F_P)

Sentence F_P shows a scope ambiguity. Either there is one person who gets mugged every ten minutes ($\varphi_{\exists\forall}$), or every ten minutes there is some person or other who gets mugged ($\varphi_{\forall\exists}$). Both states of affairs can be expressed by the more complex but unambiguous sentences F_A and F_B:

1. Every ten minutes some man or other gets mugged in New York. (F_A)
2. Every ten minutes a particular man gets mugged in New York. (F_B)

The strategic situation can be described as follows. The speaker intends to communicate $\varphi_{\exists\forall}$ or $\varphi_{\forall\exists}$ and chooses one of the sentences F_P, F_A, or F_B. If the hearer interprets the uttered sentence as intended, then communication is successful and both receive a payoff of 1. If they miscommunicate, they receive a payoff of 0. If the speaker chooses one of the unambiguous sentences, then, as they are more complex, a small amount is subtracted. In the following game tree in Figure 1, this is indicated by the minus–sign after 1. The game tree starts with the utterance of the speaker, followed by the hearer's interpretation, and finally shows their joint payoff. The hearer cannot distinguish between uttering F_P in situation $\varphi_{\forall\exists}$ and situation $\varphi_{\exists\forall}$. In Figure 1, this is indicated by the vertical line connecting the respective central nodes.

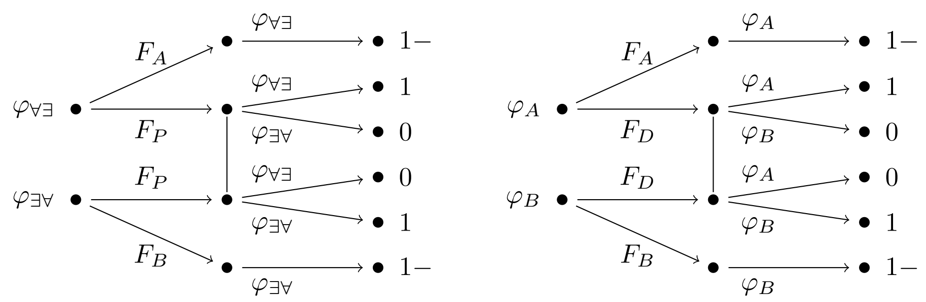


Figure 1: The game tree for Parikh's and the Doctor's Appointment example.

Speaker and hearer can follow several *strategies*. Strategies are represented by functions which map the agent's information states to actions. For example, the speaker can choose F_P in $\varphi_{\forall\exists}$, and F_B in $\varphi_{\exists\forall}$. This is represented by the function S which maps the information state $\varphi_{\forall\exists}$ to F_P, and the information state $\varphi_{\exists\forall}$ to F_B. The hearer's information state can be identified with the sentence received from the speaker. Hence, it can be F_P, F_A, or F_B. The hearer strategy maps these states to interpretations. The set of interpretations can be identified with the set of the speaker states $\varphi_{\exists\forall}$ and $\varphi_{\forall\exists}$. Parikh assumes that state $\varphi_{\forall\exists}$ is much more probable than state $\varphi_{\exists\forall}$. A strategy pair (S, H) is a *Nash equilibrium* if neither player has an interest to choose a different strategy as long as the other player sticks to his equilibrium strategy. Parikh's example shows two Nash equilibria: the two strategy pairs for which $H \circ S$ is the identity map. These are the strategy pair (S_1, H_1) for which the speaker chooses the ambiguous sentence F_P in the more probable situation $\varphi_{\forall\exists}$, and the unambiguous sentence F_B for $\varphi_{\exists\forall}$, and the strategy pair (S_2, H_2) for which the speaker chooses the ambiguous sentence F_P in the less probable situation $\varphi_{\exists\forall}$, and the unambiguous sentence F_A in $\varphi_{\forall\exists}$.[2] The two Nash equilibria are not of equal status. We call a Nash equilibrium (S, H) a *Pareto Nash* equilibrium if there is no other Nash equilibrium which is preferred by both agents. In the example, both agents have an interest to switch from (S_2, H_2) to (S_1, H_1), but they have no interest to switch from (S_1, H_1) to (S_2, H_2). This follows from the assumption that $\varphi_{\forall\exists}$ is much more probable than $\varphi_{\exists\forall}$. Hence, (S_1, H_1) is a Pareto Nash equilibrium, but not (S_2, H_2). Parikh assumes that speaker and hearer agree on Pareto Nash equilibria, from which it follows that the ambiguous sentence F_P is interpreted as *every ten minutes some man or other gets mugged in New York.*

Structurally, Parikh's example is identical to the Doctor's Appointment example:

(3) [Doctor's Appointment] Background: John is known to regularly consult two different doctors, physicians A and B. He consults A more often than B. Then, S says: '*John has a doctor's appointment at 4pm. He requests you to pick him up afterwards.*' (F_D)

After replacing F_P with F_D, and $\varphi_{\forall\exists}$ with φ_A (*John waits at A's practice*), and $\varphi_{\exists\forall}$ with φ_B (*John waits at B's practice*), we arrive at the same game tree, see Figure 1. The two examples can only differ with respect to the probabilities assigned to the two propositions the speaker intends to communicate. Parikh assumes that $\varphi_{\forall\exists}$ is much more probable than $\varphi_{\exists\forall}$. For purposes of illustration, he chooses the values $P(\varphi_{\forall\exists}) = 0.9$, and $P(\varphi_{\exists\forall}) = 0.1$ (2001, p. 28). The unique Pareto Nash equilibrium of his model does not change as long as $P(\varphi_{\forall\exists}) > P(\varphi_{\exists\forall})$. As the game trees of the two examples are structurally identical, and the equilibria only depend on one state being more probable than the other, it follows that the predictions must be identical. Whatever the probabilities of φ_A and φ_B are, as long as $P(\varphi_A) > P(\varphi_B)$, F_D must be interpreted as meaning φ_A.

The reason for the differences found in the two examples is obviously that in Parikh's example the probability of one state is not simply greater than

[2] That means, $S_1(\varphi_{\forall\exists}) = F_P$, $S_1(\varphi_{\exists\forall}) = F_B$, $H_1(F_P) = \varphi_{\forall\exists}$, $H_1(F_A) = \varphi_{\forall\exists}$, and $H_1(F_B) = \varphi_{\exists\forall}$; and $S_2(\varphi_{\exists\forall}) = F_P$, $S_2(\varphi_{\forall\exists}) = F_A$, $H_2(F_P) = \varphi_{\exists\forall}$, $H_2(F_A) = \varphi_{\forall\exists}$, and $H_2(F_B) = \varphi_{\exists\forall}$. Hence, for $\varphi = \varphi_{\forall\exists}, \varphi_{\exists\forall}$, it is $(H_1 \circ S_1)(\varphi) = (H_2 \circ S_2)(\varphi) = \varphi$.

the probability of the other state but that one state is practically certain and the other impossible. If the respective probabilities approach one or zero, it is plausible that they are subjectively treated as being identical to one or zero. However, Parikh's principle does not depend on the probabilities being close to one or zero, it makes the same predictions when they are $2/3$ and $1/3$. As the Doctor's Appointment example shows, in this case, the addressee will not accept the ambiguity but react with a clarification request.

The question of how addressees react to ambiguities is not settled without discussion of apparently similar examples like the *Out–of–Petrol* example (4), and the *Bus–Ticket* example (5):

(4) [Out of Petrol, Grice] H is standing by an obviously immobilised car and is approached by S; the following exchange takes place:
H: I am out of petrol.
S: There is a garage round the corner. (F_G)
+> The garage is open.

(5) [Bus Ticket] An email was sent to all employees that bus tickets for a joint excursion have been bought and are ready to be picked up. By mistake, no contact person was named. Hence, H asks one of the secretaries:
H: Where can I get the bus tickets for the excursion?
S: Ms. Müller is sitting in office 2.07. (F_M)
+> Bus tickets are available from Ms. Müller.

In both examples, the speaker utters a sentence which can be interpreted in two different ways, and hence involve ambiguities which are similar to the Doctor's Appointment example, as shown in Figure 2. There, φ_{opn} is the situation in which the garage is open, and $\varphi_{\neg opn}$ the situation in which it is closed; φ_{has} is the situation in which bus tickets are available from Ms. Müller, and $\varphi_{\neg has}$ the situation in which they are not. However, in none of the examples should the addressee react with a clarification request.

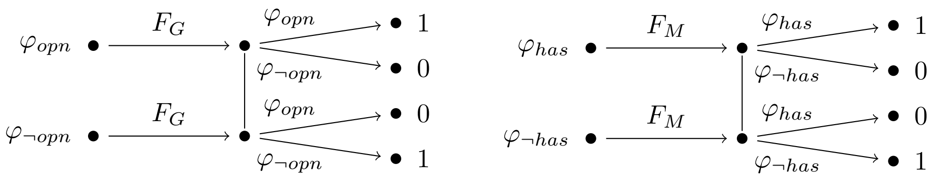


Figure 2: The Out–of–Petrol and the Bus–Ticket example.

The difference between these examples and the Doctor's Appointment example are obvious. Whereas in the Doctor's Appointment and in Parikh's example the corresponding unambiguous alternatives guarantee communicative success in both situations, this is not the case in the Out–of–Petrol and the Bus–Ticket examples. For example, if the garage round the corner is closed, then the unambiguous sentence '*There is a closed garage around the corner*' does not solve the addressee's problem of finding petrol for his car. What these examples show is that ambiguity does not automatically lead to clarification requests. Our model has to work out the precise conditions under which they lead, and under which they don't lead to clarification requests.

There is an important aspect with respect to which the Out–of–Petrol and the Bus–Ticket example differ. Whereas the pure propositional contend of the speaker's statement in the Out–of–Petrol example will lead the addressee to choose the intended action, this is not the case in the Bus–Ticket example. This can be illustrated as follows: Assume that in (4) H finds a map with all petrol stations in town and notices that (F_G) *there is a garage round the corner*. This will be sufficient information to induce him to go to this garage. Now assume that in (5) H finds a list with all the office numbers of all employees and reads there that (F_M) *Ms. Müller is sitting in office 2.07.* If there is no a priori link between F_M and Ms. Müller having bus tickets, i.e., if the two events are probabilistically independent, then learning F_M will not induce H to go to office 2.07.

We explain the Bus–Ticket example along the following lines: the speaker should have said that Ms. Müller has the bus tickets; he forgot to mention it and the addressee accommodates the missing information for making the assertion of F_M pragmatically intelligible. Hence, our model needs to include some repair mechanism which involves accommodation. This should make it also applicable to standard examples of accommodation:

(6) [Accommodation] Smith entered the room. She was wearing a red dress.

Here too, we can argue that the speaker committed an error when he used the pronoun *she* without satisfying its presupposition. The recognition of this error triggers a repair mechanism which involves accommodation. However, we will argue that the source of the error is different in the accommodation and in the Bus–Ticket example, leading to two different kinds of accommodation. Whereas in the Bus–Ticket example the speaker committed a mistake when selecting the content of his message, the speaker in (6) only committed an error when choosing the wrong form of his utterance. He could have said '*Smith was wearing a red dress*' without committing a mistake or changing the content. This distinction is systematic, and we generally distinguish between errors in message selection and errors in signal selection.

The Bus–Ticket and the Out–of–Petrol examples involve relevance implicatures. We also show that error models can explain scalar implicatures:

(7) [Scalar Implicatures] Some of the boys came to the party.

The scalar expression *some* can be used in situations in which all of the boys came, and in situations in which only some but not all came. Although this is not a semantic ambiguity, but only a case of a weak expression which could be replaced by a stronger one, it is often argued that it becomes ambiguous in situations in which the joint goal of speaker and hearer is to communicate the true state of affairs. We will argue that quantity implicatures can be explained as the result of a hearer strategy coping with speakers who omit parts of their utterances. The Bus–Ticket example and the scalar examples are structurally closely related. The difference is again explained by the fact that the Bus–Ticket example involves errors in message selection, whereas quantity implicatures origin from errors in signal selection.

3 The model of communication

Current game theoretic models of communication consider one–way communication only, i.e. communication in which the speaker sends a signal without the possibility of the hearer giving feedback to the speaker. Only if the models allow for more than one round of interactions, it is assumed that speaker and hearer can observe the outcome of communication and adjust their strategies accordingly. In this sense, they allow for feedback which involves learning about each other. In contrast, we mean by feedback the possibility to send a message back to the speaker immediately after receiving the signal, i.e. before the outcome of the game is evaluated. We only consider a simple type of feedback messages which signal the speaker that the addressee has detected an error.

As we are interested in communication with errors, a natural starting point is the communication model of Shannon & Weaver (1949). Figure 3 shows a schematic representation. It consists of five modules. First, the *information source* which produces a message to be communicated to the receiver. Second, the *transmitter* which generates a signal from the selected message. Third, the *channel* over which the signal is transmitted. Fourth, the *receiver* which reconstructs the message from the signal. Finally, fifth, the *destination*, which is the person for which the message is intended.

Shannon's theory is concerned with the question of how accurately the signal can be transmitted in the presence of *noise*. That means that during the transmission over the channel some symbols of the signal may randomly

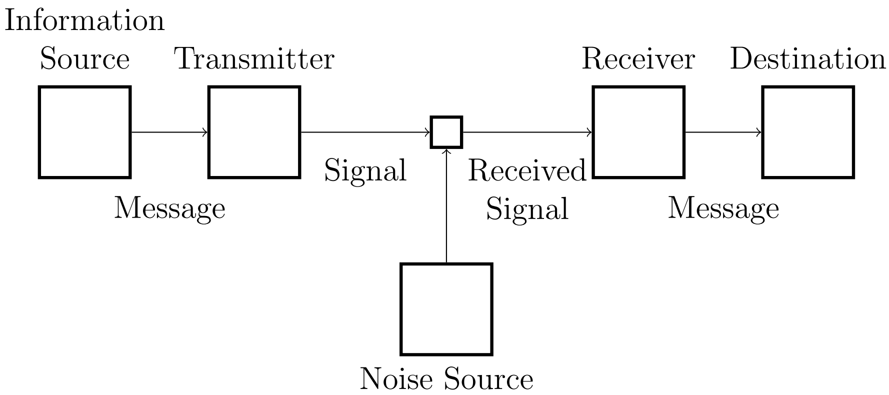


Figure 3: The Shannon–Weaver model of communication (Shannon and Weaver, 1949, p. 34).

be deleted, changed, or additional ones be added. The source of the noise is an exogenous parameter which only affects the channel of transmission.

We are concerned with the conditions under which the signal can communicate the message successfully in the presence of *errors* committed by the speaker. Errors can occur either in the selection of the message or in the generation of the signals. We will exclude the possibility of noisy channels, or of mistakes committed by the addressee. Hence, signals will always be accurately and reliably transmitted. The description of the speaker's errors will be the key parameter in the explanations of specific phenomena.

The modified model consists of four relevant modules, each representing a step in the communication process at which the speaker or the hearer makes a decision. We call the first module *message selection*. Here, the speaker selects a formula which he intends to communicate. The second step is the *signal selection* in which the speaker generates a linguistic form or sentence and sends it to the addressee. The third step is the *signal interpretation* in which the hearer reconstructs the message from the signal. Finally, the fourth step is *decision making* in which the hearer decides about subsequent actions. The fourth step is optional, and will only be considered in situations in which the joint purpose of the conversation is to solve a decision problem of the hearer. In addition, we allow for feedback messages which inform the speaker about the detection of an error. The schema of the modified model is shown in Figure 4.

If decision making is about real actions, its effect can only be evaluated with reference to the real world and the preferences of speaker and hearer. In addition, also the information states of speaker and hearer have to be

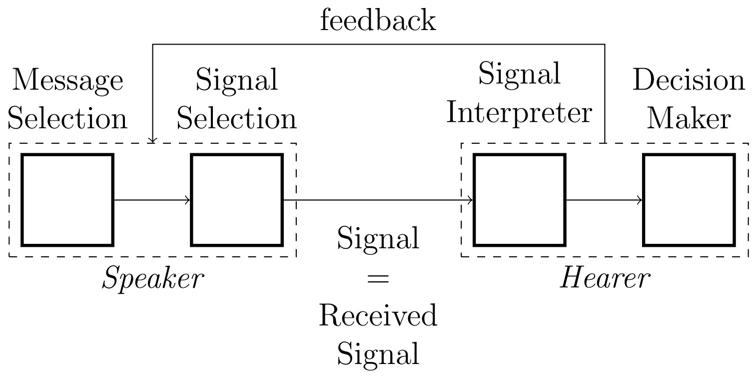


Figure 4: The modified Shannon–Weaver model with feedback loop

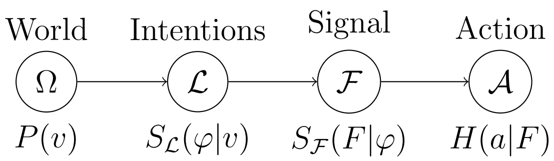
defined. We represent these parameters in *signalling games*. They describe the strategic situation in which interlocutors make decisions about signals and actions. All the examples discussed in Section 2 involve a speaker who is a domain expert. We therefore simplify our model and assume that the speaker knows the exact state of the world. We represent messages by first order formulae φ. The actual world v and intended message φ are only known to the speaker. They are his *private knowledge*. In game theory the private knowledge of an agent is called his *type*. In contrast to the speaker, we assume that the hearer has no private knowledge; this means that everything that the hearer knows is common knowledge.

We assume that interlocutors play the following variety of signalling games:

1. Nature chooses a world v from a set of possible worlds Ω with probability $P(v)$. The world v is known to the speaker alone.
2. The speaker chooses a message φ from a set $\mathcal{L}$ of first order sentences with probability $S_{\mathcal{L}}(\varphi|v)$ which depends on the world v.
3. Then he chooses a signal F from a set $\mathcal{F}$ of sentences of natural language with probability $S_{\mathcal{F}}(F|\varphi)$. The signal is reliably transmitted to the hearer.
4. The hearer interprets the signal by a formula $\varphi \in \mathcal{L}$ and chooses an action a from a set $\mathcal{A}$, or sends back a clarification request $\mathbf{c}$. If he sends a clarification request, the game starts again with message selection.
5. In the second round, no errors can occur, and the hearer can not send clarification requests. Hence, the game must end.

We distinguish between *interpretation* and *action selection* games. In an *interpretation game*, the game ends after signal interpretation. If the hearer also chooses a non–linguistic action, we call the game an *action selection game*. Furthermore, we call $S_{\mathcal{L}}(\varphi|v)$ the speaker's *message selection* strategy, and $S_{\mathcal{F}}(F|\varphi)$ his *signal selection* strategy. In contrast to the strategies considered in the previous section, these strategies are *probabilistic*, so–called *mixed* strategies. In contrast to *pure* strategies, which map information states to actions, mixed strategies select the actions only with a certain probability.[3]

In general, we will not separate the hearer's signal interpretation and decision making. If the hearer's only task is to choose an interpretation for signals, this is the case for example in models of scalar implicatures, then we represent his strategy by a probability distribution $H(\psi|F)$ over interpretations. If a decision about real actions follows, we represent his strategy by a probability distribution $H(a|F)$ over non–linguistic actions. We will speak of *actions* in general to cover both cases, and simply write $H(a|F)$ for the hearer's *interpretation strategy.* The following graph summarises the sequence of events involved in playing signalling games without clarification requests:



At the end, the outcome of the game is evaluated with respect to a shared utility measure u. That the utility measure u is shared by S and H means that there is no conflict of interest between speaker and hearer, and that they both consider the same outcomes successful. This implicitly represents the Gricean cooperative principle.

In addition to the general game structure, we make certain assumptions about the meaning of formulae and interpretation of sentences. We assume that the meaning of formulae $\varphi \in \mathcal{L}$ is described by their extension $[\![\varphi]\!] \subseteq \Omega$, and that the semantics of sentences F is described by a function $|\,.\,|$ which maps natural language sentences to non–empty sets of formulae. The set $|F|$ represents the possible translations of sentence F. If $|F|$ contains more than one formula, F is ambiguous. We assume that for all formulae φ there is a sentence F for which $|F| = \{\varphi\}$. In this case, we write $|F| = \varphi$. This condition guarantees that each formula can be expressed unambiguously.

[3]Obviously, pure strategies are a special case of mixed strategies. For Example 2 the pure strategy S_1 with $S_1(\varphi_{\forall\exists}) = F_P$ and $S_1(\varphi_{\exists\forall}) = F_B$ is equivalent to the mixed strategy S_1 with $S_1(F_P|\varphi_{\forall\exists}) = 1$ and $S_1(F_B|\varphi_{\exists\forall}) = 1$.

Finally, we have to say how *costs* of signals and clarification requests are represented. If no clarification request is sent, a branch of the game has the form $\langle v, \varphi, F, \psi, a\rangle$. If the game is an interpretation game, then the only task is to interpret the signal correctly. Hence, u has the form $\delta(\varphi, \psi) - cost(F)$ with $\delta(\varphi, \psi) = 1$ *iff* $\varphi = \psi$ and $\delta(\varphi, \psi) = 0$ else. If the game is an action selection game, we assume that the utility measure can be decomposed into the utility of performing a in v minus the costs of F: $u(v, a) - cost(F)$. This means that in an action selection game we implicitly assume that the interpretation of the signal is always successful. Furthermore, we assume that costs are *nominal*, i.e. positive but vanishingly small. In addition, we assume that costs of clarification requests are higher than the costs of unambiguously expressing the intended formula.

For Grice, the information communicated by an utterance divides into two parts, the semantic meaning of the utterance and its implicated meaning. The basic intuition about implicatures is captured by the following quote:

> "... what is implicated is what it is required that one assume a speaker to think in order to preserve the assumption that he is observing the Cooperative Principle (and perhaps some conversational maxims as well), ..." (1989, p. 86)

We generalise Grice's idea that implicatures arise from the additional information that an utterance provides about the knowledge of the speaker. More precisely, we say that an utterance of F implicates that X holds if X is certain given that F has been uttered. In decision theory, the probability of some event X given knowledge of an event Y is represented by *conditional probabilities*. For $X \subseteq \Omega$ let $P(X|F)$ be the probability of X given that the speaker uttered F, and for $\varphi \in \mathcal{L}$ let $P(\varphi|F)$ be the probability that the speaker intended to communicate φ given that he uttered F.[4] Then, we say that the utterance of F implicates that the actual world is an element of a set X, $F \mathrel{+\!\!>} v \in X$, or that it implicates that the speaker intended to communicate φ, $F \mathrel{+\!\!>} \varphi$, iff the following conditions hold:

$$F \mathrel{+\!\!>} (v \in X) \textit{ iff } P(X|F) = 1, \text{ and } F \mathrel{+\!\!>} \varphi \textit{ iff } P(\varphi|F) = 1. \tag{3.1}$$

This is a formal interpretation of Grice' idea that implicatures are what the speaker must have had in mind when making his utterance.

[4]These probabilities are defined as follows: Let $P(v, F) := P(v) \sum_\varphi S_\mathcal{L}(\varphi|v)\, S_\mathcal{F}(F|\varphi)$ '*the probability that the world is v and that the speaker utters F*,' $P(\varphi, F) := \sum_v P(v)\, S_\mathcal{L}(\varphi|v)\, S_\mathcal{F}(F|\varphi)$ '*the probability of the speaker intending to communicate* φ *and uttering F*,' and $P(F) := \sum_v P(v) \sum_\varphi S_\mathcal{L}(\varphi|v)\, S_\mathcal{F}(F|\varphi)$ '*the absolute probability of F being uttered*.' Then, the probability $P(v|F)$ that the actual world is v given the speaker uttered F, and the probability $P(\varphi|F)$ that the speaker intended to communicate φ given that he uttered F are $P(v|F) = P(v, F)/P(F)$ and $P(\varphi|F) = P(\varphi, F)/P(F)$.

4 Error models

> "The phenomenon of linguistic misunderstanding has been given very little attention in linguistics thus far. This is hard to understand, since in all sciences having to do with systems it is a well–known fact that if one wants to get insight into how a system works, it is more revealing to regard instances of small misfunctions than examples of perfect functioning." (Zaefferer, 1977, p. 329)

The situation has not changed significantly since Dietmar Zaefferer wrote this passage in the middle of the seventies. That error coping strategies should play a role in communication, and especially in pragmatics, is such an obvious idea that it makes one wonder why it has not been explored seriously before. Speakers commit errors. These errors are expected by both speaker and hearer, and this expectation is part of the common ground. *Error* models are an attempt to model the effects of such commonly expected errors in pragmatics. Errors may originate from multiple sources; hence, the representation of errors in error models is as general as possible. Errors may occur in each step of the communication process, in one single step or in several steps simultaneously. We only discuss situations in which errors occur in message selection or in signal selection, but not in both. Let $S_{\mathcal{L}}$ and $S_{\mathcal{F}}$ be two error–free speaker strategies. They define sets $\mathrm{N}_v^{\mathcal{L}} = \{\varphi \mid S_{\mathcal{L}}(\varphi|v) > 0\}$ and $\mathrm{N}_\varphi^{\mathcal{F}} = \{F \mid S_{\mathcal{F}}(F|\varphi) > 0\}$. Errors will change the strategies $S_{\mathcal{L}}$ and $S_{\mathcal{F}}$ into noisy strategies $\widetilde{S}_{\mathcal{L}}$ and $\widetilde{S}_{\mathcal{F}}$ and define sets $\mathcal{N}_v^{\mathcal{L}} = \{\varphi \mid \widetilde{S}_{\mathcal{F}}(\varphi|v) > 0\}$ and $\mathcal{N}_\varphi^{\mathcal{F}} = \{F \mid \widetilde{S}_{\mathcal{F}}(F|\varphi) > 0\}$. We call these sets *noise sets*. If there is no noise in signal selection, then $\mathcal{N}_\varphi^{\mathcal{F}} = \mathrm{N}_\varphi^{\mathcal{F}}$, and if there is no noise in message selection, then $\mathcal{N}_v^{\mathcal{L}} = \mathrm{N}_v^{\mathcal{L}}$. In both cases, we arrive at a sequence of noise sets $(\mathcal{N}_{v,\varphi})_{\langle v,\varphi\rangle \in \Omega\times\mathcal{L}}$ with

$$\mathcal{N}_{v,\varphi} = \mathcal{N}_\varphi^{\mathcal{F}} \text{ for } \varphi \in \mathcal{N}_v^{\mathcal{L}} \text{ and } \mathcal{N}_{v,\varphi} = \emptyset \text{ for } \varphi \notin \mathcal{N}_v^{\mathcal{L}}. \tag{4.2}$$

An error model is an interpreted signalling game together with a sequences of noise sets:

Definition 1 *An* error model *is a pair* $\left\langle \mathscr{G}, (\mathcal{N}_{v,\varphi})_{\langle v,\varphi\rangle\in\Omega\times\mathcal{L}} \right\rangle$ *which consists of an interpreted signalling game* $\mathscr{G} = \langle \Omega, P, \mathcal{L}, \mathcal{F}, \mathcal{A}, \mathbf{c}, [\![.]\!], |.|, u\rangle$ *and a sequence* $(\mathcal{N}_{v,\varphi})_{\langle v,\varphi\rangle\in\Omega\times\mathcal{L}}$ *of sets* $\mathcal{N}_{v,\varphi} \subseteq \mathcal{F}$.

How should the hearer react to the possibility of noise in the speaker strategy? It is here that the clarification requests come into play. According to our definition of interpreted signalling game, a clarification request **c** leads to a second round in the game. We assume that clarification requests make the speaker aware that the hearer suspects an error, and that this makes him produce a signal which is guaranteed to be free of mistakes. Together with the assumption that clarification requests only incur nominal costs, these assumptions guarantee that the hearer has a cheap means to secure an error–free answer which leads to communicative success.

It can be easily seen that the addition of efficient clarification requests in itself has no effect on the equilibria of an interpreted signalling game. This changes when we consider noisy communication.

Let $\mathscr{B}_{\mathcal{A}}(v,\varphi) = \{a \in \mathcal{A} \mid \forall b \in \mathcal{A}\; u(v,\varphi,b) \leq u(v,\varphi,a)\}$ be the set of all actions which are optimal from the speaker's perspective. After receiving signal F, the hearer knows that an action a is speaker optimal if it is an element of $\mathscr{B}_{\mathcal{A}}(v,\varphi)$ for all $\langle v,\varphi\rangle$ for which the speaker can produce F. This means that the hearer knows that an action is speaker optimal if it is an element of the set:

$$\tilde{\mathscr{B}}_{\mathcal{A}}(F) := \bigcap\{\mathscr{B}_{\mathcal{A}}(v,\varphi) \mid F \in \mathcal{N}_{v,\varphi}\}. \tag{4.3}$$

As the speaker knows the actual world and the intended message, the speaker optimal actions are the objectively optimal actions. Hence, if $\tilde{\mathscr{B}}_{\mathcal{A}}(F)$ is not empty, then the hearer can safely choose any action from it. If it is empty, then he better chooses a clarification request. The reason is that $\tilde{\mathscr{B}}_{\mathcal{A}}(F) = \emptyset$ implies that for every action a there is a speaker type $\langle v,\varphi\rangle$ for which $F \in \mathcal{N}_{v,\varphi}$ and for which a is not optimal. This would make the choice of any action risky. Hence, we assume that the *canonical* hearer strategy $\overline{H}$ which takes into account the noise described by an error model is such that it reacts to signals F with clarification request **c** if $\tilde{\mathscr{B}}_{\mathcal{A}}(F) = \emptyset$, and otherwise chooses any of the actions in $\tilde{\mathscr{B}}_{\mathcal{A}}(F)$ with equal probability.

For this new strategy, the speaker has the possibility to improve his original strategy S by intentionally choosing signals which otherwise could only be produced under the influence of noise. Let $\mathcal{U}$ be the set of all $F \in \mathcal{F}$ for which $\tilde{\mathscr{B}}_{\mathcal{A}}(F)$ is not empty:

$$\mathcal{U} := \{F \in \mathcal{F} \mid \tilde{\mathscr{B}}_{\mathcal{A}}(F) \neq \emptyset\}. \tag{4.4}$$

Then, the speaker can choose from $\mathcal{N}_{v,\varphi} \cap \mathcal{U}$. Let $\mathcal{U}_{v,\varphi}$ be the minimally complex signals from $\mathcal{N}_{v,\varphi} \cap \mathcal{U}$. If $\mathcal{N}_{v,\varphi} \cap \mathcal{U} \neq \emptyset$, then the speaker will choose some element from $\mathcal{U}_{v,\varphi}$. If $\mathcal{N}_{v,\varphi} \cap \mathcal{U}$ is empty, the speaker can send any signal in $\mathcal{N}_{v,\varphi}$. The hearer will react to this with a clarification request, and the next round guarantees an optimal outcome. Let $\overline{S}$ be any speaker strategy of this sort, then there is no other strategy pair which would provide higher payoffs except for the nominal costs of signals.

5 Errors in message selection

In this section we consider the effects of errors which occur in message selection. All examples are examples of action selection games. The Out–of–Petrol example, the Bus–Ticket example, and the Doctor's Appointment example belong here.

We assume here that error–free games are solved such that the hearer chooses actions which he expects to be optimal given the semantic meaning of the speaker's signal, and that the speaker chooses the least costly signal which unambiguously expresses a formula which leads the hearer to choose a speaker optimal action.[5] If the speaker utters F, we write $\mathscr{B}_{\mathcal{A}}(F)$ for the set of actions the hearer believes to be optimal.

We first consider the Bus–Ticket example (5). The possible worlds differ according to the contact person and his office number. To simplify the model, we assume that there are exactly two staff from whom bus tickets may be available, Ms. Müller m and Mr. Schmidt s. Furthermore, we assume that the tickets are available from Ms. Müller iff they are not available from Mr. Schmidt, and that one of them is sitting in office 2.07 iff the other one is sitting in 3.11. Hence, the model only contains four worlds and two actions. We assume that all possibilities are equally probable. Let φ_i be the formula stating that i has the tickets, and $\varphi_{i/n}$ the formula stating that i is sitting in office n; let F_i and $F_{i/n}$ be the respective natural sentences which express φ_i and $\varphi_{i/n}$. Finally, let *go-to-n* be the act of going to office number n. The utilities are shown in the following table. The last column contains the formulae which the speaker must communicate for informing the addressee about the exact state of the world:

Ω	φ_m	$\varphi_{m/2.07}$	*go-to-2.07*	*go-to-3.11*	
v_1	+	+	1	0	$\varphi_m \wedge \varphi_{m/2.07}$
v_2	+	−	0	1	$\varphi_m \wedge \varphi_{m/3.11}$
v_3	−	+	0	1	$\varphi_s \wedge \varphi_{s/3.11}$
v_4	−	−	1	0	$\varphi_s \wedge \varphi_{s/2.07}$

We assume that the cheapest sentences expressing φ_i and $\varphi_{i/n}$ are respectively F_i '*Bus tickets are available from i*' and $F_{i/n}$ '*i is sitting in office n.*' First, we show that the sentences $F_i \wedge F_{i/n}$ are optimal for the noise free game, but none of the conjuncts $F_i, F_{i/n}$. As explained above, we assume that in the noise–free game the hearer chooses the action which he thinks to be optimal given the semantic content of the utterance. If the speaker says F_i

[5] A full justification of this approach is beyond the scope of this paper, but see (Benz, 2006). This assumption does not affect the predictions of the noise sets, but it is necessary for justifying the noise sets.

or $F_{i/n}$, the hearer learns from their semantics that the actual world belongs to $[\![\varphi_i]\!]$ or $[\![\varphi_{i/n}]\!]$ respectively. Hence, he can not decide whether *go-to-2.07* or *go-to-3.11* is the better choice. In contrast, if the speaker utters $F_i \wedge F_{i/n}$, then the hearer knows that *go-to-n* is the best action. Hence, $\mathscr{B}_{\mathcal{A}}(F_i) = \mathscr{B}_{\mathcal{A}}(F_{i/n}) = \{\textit{go-to-2.07}, \textit{go-to-3.11}\}$, and $\mathscr{B}_{\mathcal{A}}(F_i \wedge F_{i/n}) = \{\textit{go-to-2.07}\}$. This entails that the speaker's best choice is to communicate the formula $\varphi_i \wedge \varphi_{i/n}$ which is true in the actual world, and hence to utter its respective literal expression $F_i \wedge F_{i/n}$. As in Section 4, let $\mathrm{N}_v^{\mathcal{L}}$ denote the set of all formulae which can be chosen by the speaker's error–free message selection strategy. The elements of $\mathrm{N}_v^{\mathcal{L}}$ are then the formulae shown in the last column of the previous table.

The following definition of noise sets captures the intuition that the secretary in the Bus–Ticket example simply forgot to name a conjunct of the optimal message. The noise sets only depend on the actual world v. We set:

$$\mathcal{N}_v^{\mathcal{L}} = \{\psi \mid v \in [\![\psi]\!] \wedge \exists \varphi \in \mathrm{N}_v^{\mathcal{L}}\ \psi \lhd \varphi\}. \tag{5.5}$$

Here $\psi \lhd \varphi$ is a transitive relation which says that $\psi = \varphi$, or ψ is a conjunct of φ; i.e. $\varphi \lhd \varphi$, and $\varphi, \psi \lhd \varphi \wedge \psi$. Hence, we arrive at the noise sets $\mathcal{N}_v^{\mathcal{L}}$ which are the union of the formulae shown in the last two colums:

Ω	φ_m	$\varphi_{m/2.07}$	$\mathrm{N}_{v_i}^{\mathcal{L}}$	$\mathcal{N}_{v_i}^{\mathcal{L}} \setminus \mathrm{N}_{v_i}^{\mathcal{L}}$
v_1	+	+	$\varphi_m \wedge \varphi_{m/2.07}$	$\varphi_m,\ \varphi_{m/2.07}$
v_2	+	−	$\varphi_m \wedge \varphi_{m/3.11}$	$\varphi_m,\ \varphi_{m/3.11}$
v_3	−	+	$\varphi_s \wedge \varphi_{s/3.11}$	$\varphi_s,\ \varphi_{s/3.11}$
v_4	−	−	$\varphi_s \wedge \varphi_{s/2.07}$	$\varphi_s,\ \varphi_{s/2.07}$

We can read off the implicatures ψ of $F_{m/2.07}$ '*Ms. Müller is sitting in office* 2.07' from the rows which contain $\varphi_{m/2.07}$. If there is an act which is optimal in all rows containing $\varphi_{m/2.07}$, then $\tilde{\mathscr{B}}_{\mathcal{A}}(F_{m/2.07})$ is not empty, and $F_{m/2.07}$ is not answered by a clarification request. Furthermore, if ψ is true in all worlds belonging to a row containing $\varphi_{m/2.07}$, then $F_{m/2.07}$ implicates ψ. This graphical solution can be justified as follows. As the rows represent the possible branches of the underlying signalling game, the fact that the actual world v is an element of $[\![\psi]\!]$ for all branches in which $F_{m/2.07}$ can be uttered immediately implies that $P([\![\psi]\!]|F_{m/2.07}) = 1$. By definition (3.1), this is equivalent to $F_{m/2.07} +\!> v \in [\![\psi]\!]$. As the speaker's signal selection strategy is free of errors and follows the semantic convention, $F_{m/2.07}$ can only be chosen as verbalisation of $\varphi_{m/2.07}$. The formula $\varphi_{m/2.07}$ in turn can only occur as element of $\mathcal{N}_{v_1}^{\mathcal{L}}$. As $v_1 \models \varphi_m$ holds, it follows that $P([\![\varphi_m]\!]|F_{m/2.07}) = 1$, and hence that $F_{m/2.07}$ implicates that φ_m '*Ms. Müller has the bus tickets*'. $P(v_1|F_{m/2.07}) = 1$ also implies that $\tilde{\mathscr{B}}_{\mathcal{A}}(F_{m/2.07}) = \{\textit{go-to-2.07}\}$ is not empty, and therefore that the hearer will not react with a clarification request but choose the intended action. In contrast, if the secretary had said F_m '*Ms. Müller has the bust tickets*', the model would predict a clarification request. This follows from $\tilde{\mathscr{B}}_{\mathcal{A}}(F_m) = \emptyset$, which in turn follows from the fact that F_m can occur in both v_1 and v_2, and the fact that there is no action which is optimal in both worlds.

We next compare the Bus–Ticket with the Out–of–Petrol example (4). We distinguish four equi-probable worlds $\{v_1, v_2, v_3, v_4\}$ and two actions: *go-to-d*, where d is the place of the garage, and a random search r. Let $\varphi_G = |F_G|$ be the formula which states that d is a petrol station, and φ_{opn} that the place is open. The worlds and utilities are defined as follows:

Ω	φ_G	φ_{opn}	*go-to-d*	r
v_1	+	+	1	ε
v_2	+	−	0	ε
v_3	−	+	0	ε
v_4	−	−	0	ε

The answering expert knows that he is in v_1. We assume that ε is such that after learning φ_G the hearer prefers *go-to-d* over the random search. This means that $\mathscr{B}_{\mathcal{A}}(F_G) = \{go\text{-}to\text{-}d\}$, and therefore that $\varphi_G \in \mathrm{N}^{\mathcal{L}}_{v_1}$. This marks the difference between the Out–of–Petrol and the Bus–Ticket example. The consideration of error sets is not necessary for calculating the implicature. Although the trees shown in Figure 2 seem to be structurally identical, the situation is quite different. In the Out–of–Petrol example no accommodation is necessary. This is in agreement with the intuition that the interpretation of the answer in the Bus–Ticket example needs more effort than in the Out–of–Petrol example. [6]

Finally, we consider the Doctor's Appointment example. We consider a model with two worlds, a world v_1 in which John is waiting at A's practice, and a world v_2 in which he is waiting at B's practice. Let us assume that $P(v_1) = 2/3$, and $P(v_2) = 1/3$. The speaker said F_D: '*John has a doctor's appointment at 4pm. He requests you to pick him up afterwards.*' His assertion would have been unmistakable if he had said (F_A) '*John has a doctor's appointment at A's practice at 4pm. He requests you to pick him up afterwards*' if John waits at A's practice, or (F_B) '*John has a doctor's appointment at B's practice at 4pm. He requests you to pick him up afterwards*' if he waits

[6]The difference does not depend on the fact that we considered only one alternative in the Out–of–Petrol but two in the Bus–Ticket example. In (Benz, 2011) the type of models used for the Out–of–Petrol example were called *normal optimal answer* models. This paper also provides a procedure for their construction and a detailed justification.

at B's practice. Let $|F_A| = \varphi_A$, $|F_B| = \varphi_B$, and $|F_D| = \varphi_D$. As φ_A is more probable than φ_B, we find that $\mathscr{B}_{\mathcal{A}}(F_A) = \mathscr{B}_{\mathcal{A}}(F_D) = \{\textit{go-to-A}\}$, and $\mathscr{B}_{\mathcal{A}}(F_B) = \{\textit{go-to-B}\}$. Taking complexity into account, it follows that in the error–free game the speaker should say F_D in v_1 and F_B in v_2. This explains the following table:

Ω	φ_A	φ_B	*go-to-A*	*go-to-B*	$\mathrm{N}^{\mathcal{L}}_{v_i}$	$\mathcal{N}^{\mathcal{L}}_{v_i}$
v_1	+	−	1	0	φ_D	φ_D
v_2	−	+	0	1	φ_B	φ_B, φ_D

Hence, for noise–free communication, we arrive at the same prediction as Prashant Parikh: the speaker should use the ambiguous but less complex signal F_D in the frequent situation, and the complex but unambiguous signal F_B in the less frequent situation. This changes when we take errors into account. φ_D is a conjunct of φ_A and φ_B. Hence, assuming the same noise set as in (5.5), $\varphi_D \in \mathcal{N}^{\mathcal{L}}_{v_1} \cap \mathcal{N}^{\mathcal{L}}_{v_2}$, and therefore $F_D \in \mathcal{N}_{v_1,\varphi_A} \cap \mathcal{N}_{v_2,\varphi_B}$. It follows that $\tilde{\mathscr{B}}_{\mathcal{A}}(F_D) = \emptyset$, and our model predicts that the hearer reacts with a clarification request. This example shows that taking errors into account is essential to pragmatics.

6 Errors in signal selection

In this section, we consider the effects of errors occurring in signal selection. This means, the speaker has chosen an appropriate message but misses the conventionally correct signal. In this section, all games are interpretation games. We show how to apply error models to quantity implicatures and presupposition accommodation.

By definition of an interpretation game, the objectively best action for the hearer in a situation in which the speaker intends to communicate a formula φ is to choose φ. When taking errors into account, choosing φ as interpretation of F is the best response if φ is the unique element of:

$$\tilde{\mathscr{B}}_{\mathcal{A}}(F) := \bigcap \{\{\varphi\} \mid \exists v : \ F \in \mathcal{N}_{v,\varphi}\}. \tag{6.6}$$

If $\tilde{\mathscr{B}}_{\mathcal{A}}(F)$ is empty, then the hearer cannot be sure how to interpret F. Hence, a clarification request is the best response. We will use (6.6) for explaining quantity implicatures and presupposition accommodation.

6.1 Quantity implicatures

Implicatures of complex sentences have been intensively discussed over the last years. One of the most important contributions to this discussion is

(Sauerland, 2004) who considers, among others, examples of the following type:

(8) **a)** Kai had some of the broccoli.
+> Kai didn't have all of the broccoli.

b) Some of the people ate some of their broccoli.
+> Some but not all of the people ate some but not all of their broccoli.

In this section we show how error models can be applied to these examples. We assume that the relevant set of signals $\mathcal{F}$ are sentences of the form: '*Kai had the broccoli,*' '*Kai had some of the broccoli,*' '*Kai had all of the broccoli,*' and connections built from these sentences with the help of '*and,*' '*or,*' '*it is not the case that...*'. In addition, we may consider sentences of the form '*Some of the people had some of the broccoli,*' '*All of the people had some of the broccoli,*' etc.

We first consider Example (8a). We use suggestive notation and write e.g. $F_\forall$ for '*Kai had all of the broccoli,*' $F_{\exists\neg\forall}$ for '*Kai had some but not all of the broccoli,*' $F_{\neg\exists}$ for '*Kai had none of the broccoli,*' etc. Accordingly, we denote the corresponding logical formulae by $\varphi_\forall$, $\varphi_{\exists\neg\forall}$, $\varphi_{\neg\exists}$, etc. About the costs we only need to assume that $cost(F_x) < cost(F_y)$ iff F_y is intuitively more complex than F_x.

We explain quantity implicatures as the effect of the speaker's tendency to omit parts of the sentence which would literally expresses the intended message. More specifically, we assume that the speaker may only omit some *conjunct* of the optimal signal. Let $F \lhd G$ hold, iff F results from G by (possibly) omitting some conjunct of G. The following relations hold:

$$F_\exists, F_{\neg\forall}, F_{\exists\neg\forall} \lhd F_{\exists\neg\forall}, \text{ and } F_\exists, F_{\neg\forall}, F_\exists \wedge F_{\neg\forall} \lhd F_\exists \wedge F_{\neg\forall}. \tag{6.7}$$

The definition implies that if F expresses ψ, and G expresses φ, then $F \lhd G$ implies $\varphi \rightarrow \psi$. We assume that the errors only depend on the formula which the speaker intends to communicate but not on the state of the world. Therefore, we can leave the actual state of the world out of consideration most of the time. The only restriction imposed by v is that the intended message φ must be true in v. Let $Lit(\varphi)$ be the set of the cheapest signals literally expressing formula φ. Then, the speaker's tendency to omit conjuncts of the literal expressions produces the following noise set:

$$\mathcal{N}_\varphi = \{F \mid \exists G \in Lit(\varphi)\; F \lhd G\}. \tag{6.8}$$

For (8a) we can assume that the speaker may only have the intention to express $\varphi_{\neg\exists}$, $\varphi_{\exists\neg\forall}$, or $\varphi_\forall$. A speaker following the literal strategy can only choose signals $F_{\neg\exists}$, $F_{\exists\neg\forall}$, or $F_{\forall}$. It is $\mathcal{N}_{\varphi_{\neg\exists}} = \{F_{\neg\exists}\}$, $\mathcal{N}_{\varphi_{\exists\neg\forall}} = \{F_{\exists\neg\forall}, F_{\exists}, F_{\neg\forall}\}$, and $\mathcal{N}_{\varphi_{\forall}} = \{F_{\forall}\}$. In Section 4, we introduced $\mathcal{U}_{\varphi}$ as the set of all minimally complex $F \in \mathcal{N}_{\varphi}$ for which $\tilde{\mathscr{B}}_{\mathcal{A}}(F) \neq \emptyset$. Using a table as before, we arrive at the following situation:

formula φ	$Lit(\varphi)$	$\mathcal{N}_{\varphi}$	$\mathcal{U}_{\varphi}$
$\varphi_{\neg\exists}$	$F_{\neg\exists}$	$F_{\neg\exists}$	$F_{\neg\exists}$
$\varphi_{\exists\neg\forall}$	$F_{\exists\neg\forall}$	$F_{\exists\neg\forall}, F_{\exists}, F_{\neg\forall}$	$F_{\exists}$
$\varphi_{\forall}$	$F_{\forall}$	$F_{\forall}$	$F_{\forall}$

The table immediately shows that $P(\varphi_{\exists\neg\forall}|F_{\exists}) = 1$, i.e. that it is certain that the speaker intended to communicate $\varphi_{\exists\neg\forall}$ given that he uttered $F_{\exists}$. By Definition 3.1, this means that $F_{\exists} +\!> \varphi_{\exists\neg\forall}$.

Next, we consider Example (8b) '*Some of the people ate some of their broccoli*' ($F_{\exists|\exists}$). We write $\exists^!$ short for *some but not all*, and $\emptyset$ for *none*. We find the following formulae and signals:

$$\begin{array}{|c|ccc|}\hline & \emptyset & \exists^! & \forall \\ \hline \emptyset & \varphi_{\emptyset|\emptyset} & \varphi_{\emptyset|\exists^!} & \varphi_{\emptyset|\forall} \\ \exists^! & \varphi_{\exists^!|\emptyset} & \varphi_{\exists^!|\exists^!} & \varphi_{\exists^!|\forall} \\ \forall & \varphi_{\forall|\emptyset} & \varphi_{\forall|\exists^!} & \varphi_{\forall|\forall} \\ \hline \end{array} \qquad \begin{array}{|c|ccc|}\hline & \emptyset & \exists^! & \forall \\ \hline \emptyset & F_{\emptyset|\emptyset} & F_{\emptyset|\exists^!} & F_{\emptyset|\forall} \\ \exists^! & F_{\exists^!|\emptyset} & F_{\exists^!|\exists^!} & F_{\exists^!|\forall} \\ \forall & F_{\forall|\emptyset} & F_{\forall|\exists^!} & F_{\forall|\forall} \\ \hline \end{array} \tag{6.9}$$

We assume that the quantifier $\exists^!$ is the conjunction '*some but not all*', i.e. $\exists \wedge \neg\forall$. Due to the possessive pronoun, the first quantifier always takes scope over the second quantifier. Hence, $|F_{\exists|\exists}| = \varphi_{\exists|\exists}$, $|F_{\exists|\forall}| = \varphi_{\exists|\forall}$, $|F_{\exists^!|\forall}| = \varphi_{\exists^!|\forall}$, etc. As the second quantifier appears in the scope of the first quantifier, we have to adjust the definition of sub–signal $\lhd$ such that it is guaranteed that the respective sub–formula is implied by the original formula. Hence, we say that $F_{P'|Q'} \lhd F_{P|Q}$ iff P' is identical to or a conjunct of P and Q' is identical to or a conjunct of Q such that $\varphi_{P|Q} \to \varphi_{P'|Q'}$. This means for example:

$$F_{\emptyset|\forall} \wedge F_{\forall|\exists}, F_{\emptyset|\forall}, F_{\forall|\exists} \lhd F_{\emptyset|\forall} \wedge F_{\forall|\exists}, \quad F_{\exists^!|\exists^!}, F_{\exists|\exists^!}, F_{\exists|\exists} \lhd F_{\exists^!|\exists^!}. \tag{6.10}$$

However, $F_{\exists^!|\exists} \ntriangleleft F_{\exists^!|\exists^!}$, and $F_{\emptyset|\exists} \ntriangleleft F_{\emptyset|\exists^!}$. The noise sets remain unchanged, i.e. $\mathcal{N}_{\varphi} = \{F \mid \exists G \in Lit(\varphi)\ F \lhd G\}$.

We consider the situation in which it is common knowledge that the speaker does not intend to fully characterise the state of the world but chooses a messages of the form $\varphi_{\forall|\forall}, \varphi_{\forall|\exists^!}, \varphi_{\forall|\emptyset}, \varphi_{\exists^!|\exists^!}$, etc. In particular, we assume that the speaker does not intend to communicate stronger messages as e.g. '*Tom had half of his broccoli, and Mary had two spoons of hers.*'

The first column of Table 1 shows all possibilities. The second column shows the noise sets, and the third the speaker optimal signal as predicted by

formula φ	$\mathcal{N}_\varphi$	$\mathcal{U}_\varphi$
$\varphi_{\forall\|\forall}$	$F_{\forall\|\forall}$	$F_{\forall\|\forall}$
$\varphi_{\forall\|\exists!}$	$F_{\forall\|\exists!}, F_{\forall\|\exists}$	$F_{\forall\|\exists}$
$\varphi_{\forall\|\emptyset}$	$F_{\forall\|\emptyset}$	$F_{\forall\|\emptyset}$
$\varphi_{\exists!\|\forall}$	$F_{\exists!\|\forall}, F_{\exists\|\forall}$	$F_{\exists\|\forall}$
$\varphi_{\emptyset\|\forall}$	$F_{\emptyset\|\forall}$	$F_{\emptyset\|\forall}$
$\varphi_{\exists!\|\exists!}$	$F_{\exists!\|\exists!}, F_{\exists\|\exists!}, F_{\exists\|\exists}$	$F_{\exists\|\exists}$
$\varphi_{\emptyset\|\exists!}$	$F_{\emptyset\|\exists!}$	$F_{\emptyset\|\exists!}$
$\varphi_{\exists!\|\emptyset}$	$F_{\exists!\|\emptyset}, F_{\exists\|\emptyset}$	$F_{\exists\|\emptyset}$
$\varphi_{\emptyset\|\emptyset}$	$F_{\emptyset\|\emptyset}$	$F_{\emptyset\|\emptyset}$

Table 1: *Some of the people had some of their broccoli.*

the error model. We can see from the sixth line that $F_{\exists|\exists}$ '*Some of the people had some of their broccoli*' implicates that some but not all of the people had some but not all of their broccoli ($\varphi_{\exists!|\exists!}$). $F_{\exists|\exists!}$ is excluded because $\mathcal{U}_\varphi$ collects signals of minimal complexity only; otherwise we would also get the implicature $F_{\exists|\exists!}$ +> $\varphi_{\exists!|\exists!}$. We also see that $F_{\forall|\exists}$ '*All of the people had some of their broccoli*' implicates that all of the people had some but not all of their broccoli ($\varphi_{\forall|\exists!}$).

6.2 Accommodation

Accommodation arguably also involves a violation of the rule of proper usage of language. If the *pre* in presupposition is interpreted as meaning that the presupposed information must be satisfied in the common ground before the presupposing expression is used, then presupposition accommodation is a repair strategy which the speaker can intentionally exploit to shorten his utterances. The following example shows three cases involving a pronoun:

(9) **a)** Maria entered the room. She was in a hurry.

b) Smith entered the room. She was in a hurry.

c) Maria entered the room and Magdalena left it. She was in a hurry.

The pronoun *she* presupposes that the common ground contains a unique female referent. In (9a), this presupposition is satisfied. In (9b) it is not satisfied but the situation can be repaired by accommodation of *Smith is female*. In (9c), there are two female persons in the common ground, and hence the interpretation of the pronoun becomes ambiguous.

We explain (9b) and (9c) by errors occurring in signal selection, i.e. the speaker chose an admissible message but did not choose the appropriate linguistic form. For example, the speaker could have replaced the pronoun *she* by the proper names of the intended person in (9b) and (9c). As a consequence, we assume that presuppositions are carried by linguistic expressions and not by the logical forms themselves.[7] We write $F\!:\!\varphi$ if signal F presupposes φ. For simplicity, we represent the common ground by a DRS (Kamp and Reyle, 1993) and make it part of the actual world. This means, we model *worlds* as pairs of DRSes $\langle D, C\rangle$ in which C represents the common ground, and D contains all the knowledge of the speaker. A DRS is a pair $\langle U, \mathit{Cond}\rangle$ consisting of a set U of variables, and a set Cond of first order formulae for which all free variables are elements of U. The elements of U are called *discourse referents.* If $\langle D, C\rangle$ is a world, then we assume that $U^C \subseteq U^D$ and $\mathit{Cond}^C \subseteq \mathit{Cond}^D$. '*Maria entered the room*' can be represented by the DRS $\langle\{x, y\}, \{\mathit{Maria}(x), \mathit{room}(y), \mathit{entered}(x, y)\}\rangle$. As *Maria* is a female name, we assume that $\mathit{female}(x)$ is automatically added. For example (9) this leads to the following representations of the common ground:

- $C_a = \langle\{x, y\}, \{\mathit{Maria}(x), \mathit{female}(x), \mathit{room}(y), \mathit{entered}(x, y)\}\rangle$;
- $C_b = \langle\{x, y\}, \{\mathit{Smith}(x), \mathit{room}(y), \mathit{entered}(x, y)\}\rangle$;
- $C_c = \langle\{x, y, z\}, \{\mathit{Maria}(x), \mathit{female}(x), \mathit{room}(y), \mathit{entered}(x, y), \mathit{Magdalena}(z), \mathit{female}(z), \mathit{left}(z, y)\}\rangle$;

We represent the fact that the pronoun *she* presupposes a female antecedent by $\mathit{she}\!:\!\mathit{female}(x)$. For our example, it suffices to assume that the speaker's private DRS specifies for each discourse referent whether the person the referent refers to is male or female. This means that the speaker's DRS D_α is constructed by adding the conditions $\mathit{female}(r)$ or $\mathit{male}(r)$ to C_α. We arrive at: $\mathit{Cond}^{D_a} = \mathit{Cond}^{C_a}$, $\mathit{Cond}^{D_b^1} = \mathit{Cond}^{C_b} \cup \{\mathit{male}(x)\}$, $\mathit{Cond}^{D_b^2} = \mathit{Cond}^{C_b} \cup \{\mathit{female}(x)\}$, $\mathit{Cond}^{D_c} = \mathit{Cond}^{C_c}$. With these preliminaries, we can formulate the noise set for accommodation:

$$\mathcal{N}_{\langle D,C\rangle,\varphi} = \{F\!:\!\psi \mid |F| = \varphi \text{ and } \psi \in \mathit{Cond}^D\}. \tag{6.11}$$

This means that the speaker may use a signal F which presupposes ψ in a situation in which he intends to communicate φ if he privately knows that ψ is satisfied.

If r is the discourse referent the speaker intendeds to refer to, then the intended message is $\varphi_r = \mathit{was\text{-}in\text{-}hurry}(r)$. For proper names we assume

[7] This is a digression from common representations, see e.g. the overview in (Beaver, 1997).

that they don't carry presuppositions. The second column of the following table shows the forms which the speaker could have used without violating presuppositions, and the third column the respective noise sets. For the last column, it is assumed that pronouns are preferred over proper names. Discourse referent x refers to Maria in the first and fourth row, and to Smith in the second and third. In the fifth row, discourse referent z refers to Magdalena. This leads to:

$v = \langle D, C\rangle$	φ	$Lit(v, \varphi)$	$\mathcal{N}_{v,\varphi}$	$\mathcal{U}_{v,\varphi}$
$\langle D_a, C_a\rangle$	φ_x	Maria, she	Maria, she	she
$\langle D_b^1, C_b\rangle$	φ_x	Smith	Smith, he	he
$\langle D_b^2, C_b\rangle$	φ_x	Smith	Smith, she	she
$\langle D_c, C_c\rangle$	φ_x	Maria	Maria, she	Maria
$\langle D_c, C_c\rangle$	φ_z	Magdalena	Magdalena, she	Magdalena

For example (9a), only the first row is relevant. As there is only one possible world, and only one possible intention, it trivially follows that the hearer will choose Maria as referent for *she*. For example (9b), the second and third row are relevant. As there can only be one intended referent, the hearer can infer that the actual world is $\langle D_b^2, C_b\rangle$, which means that Smith is female and the referent of *she*. For example (9c), the fourth and the fifth row are relevant. Here we find two possible intended referents. As the hearer cannot decide which interpretation is correct, he should react with a clarification request. This explains the differences between (9a), (9b), and (9c).

As in the case of quantity implicatures, we can only provide an impression of how to use error models for modelling presupposition accommodation. This closes our discussion of errors in signal selection.

7 Conclusions

Our aim was to show that error coping strategies play an essential role in linguistic pragmatics. For this, we tested our model by a number of benchmark examples, including examples of relevance implicatures, quantity implicatures, and presupposition accommodation. Important inspiration came from the Shannon–Weaver model of communication.

We want to highlight some of the differences between error models presented here and previous formal accounts. Of course, the most obvious difference is the role of errors. In addition, we may mention:

1. Error models are based on a communication model with feedback. This rules out Parikh's principle, but it also marks a difference to other approaches as e.g. the optimal answer approach (Benz and van Rooij,

2007), iterated best response models (Franke, 2009; Jäger and Ebert, 2009), and online interpretations of many variants of optimality theoretic pragmatics (Blutner, 2000).

2. Costs of signals are assumed to be nominal. This entails that the speaker cannot prefer imprecise signals over precise signals simply on the grounds that the latter are more costly. This rules out blocking due to high signalling costs.

3. There are no scales. The set of alternatives from which the speaker could have chosen is the set of *all* signals with a similar level of precision. This marks a difference to Neo–Gricean approaches, and in particular to (Franke, 2009; Sauerland, 2004).

4. Error models provide a uniform framework in which relevance and quantity implicatures can be explained, as well as presupposition accommodation.

As we could treat the most basic cases of relevance implicatures, scalar implicatures and accommodation only, these claims are in need of further justification which can only be provided by more detailed studies. These studies will also necessitate a number of extensions of the models. For example, a possibility to handle implicature cancellation, a cognitively plausible model of message and signal selection, as well as a combination of errors in both selection processes. But, in spite of many open issues, we are convinced that error models have a promising future.

References

Beaver, D. (1997). Presupposition. In van Benthem, J. and ter Meulen, A., editors, *Handbook of Logic & Language*, pages 939–1008. Elsevier, Amsterdam.

Benz, A. (2006). Utility and relevance of answers. In Benz, A., Jäger, G., and van Rooij, R., editors, *Game Theory and Pragmatics*, pages 195–214. Palgrave Macmillan, Basingstoke.

Benz, A. (2011). How to set up normal optimal answer models. In Benz, A., Ebert, C., Jäger, G., and van Rooij, R., editors, *Language, Games, and Evolution*, volume 6207 of *Lecture Notes in Artificial Intelligence*, pages 14–39. Springer, Heidelberg.

Benz, A. and van Rooij, R. (2007). Optimal assertions and what they implicate: a uniform game theoretic approach. *Topoi - an International Review of Philosophy*, 27(1):63–78.

Blutner, R. (2000). Some aspects of optimality in natural language interpretation. *Journal of Semantics*, 17:189–216.

Clark, R. (2009). Meaningful games: Exploring language with game theory. Book Ms., University of Pennsylvania.

Franke, M. (2009). *Signal to Act: Game Theory in Pragmatics*. PhD thesis, Universiteit van Amsterdam. ILLC Dissertation Series DS-2009-11.

Grice, H. P. (1989). *Studies in the Way of Words*. Harvard University Press, Cambridge MA.

Jäger, G. and Ebert, C. (2009). Pragmatic rationalizability. In Riester, A. and Solstad, T., editors, *Proceedings of Sinn und Bedeutung*, volume 13, pages 1–15.

Kamp, H. and Reyle, U. (1993). *From Discourse to Logic. Introduction to Modeltheoretic Semantics of Natural Language, Formal Logic and Discourse Representation Theory*. Kluwer, Dordrecht.

Parikh, P. (2001). *The Use of Language*. CSLI Publications, Stanford.

Parikh, P. (2006). Pragmatics and games of partial information. In Benz, A., Jäger, G., and van Rooij, R., editors, *Game Theory and Pragmatics*, pages 101–121. Palgrave Macmillan, Basingstoke.

Parikh, P. (2010). *Language and Equilibrium*. The MIT Press, Cambridge, MA.

Ross, I. (2006). *Games Interlocutors Play: New Adventures in Compositionality and Conversational Implicature*. PhD thesis, University of Pennsylvania.

Sauerland, U. (2004). Scalar implicatures in complex sentences. *Linguistics and Philosophy*, 27:367–391.

Shannon, C. E. and Weaver, W. (1949). *The Mathematical Theory of Communication*. University of Illinois Press, Urbana, IL.

Zaefferer, D. (1977). Understanding misunderstandings: A proposal for an explanation of reading choices. *Journal of Pragmatics*, 1:329–346.