Implicature of Complex Sentences in Error Models

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Due to their intensive discussion, implicatures of complex sentences became a kind of benchmark for testing different frameworks of Gricean pragmatics. We propose a novel approach which is based on a communication model with feedback and speaker errors in signal selection. The communication model is introduced as an extension of standard signalling games. In this model, implicatures are explained by the speaker's tendency to omit parts of their utterances. In this explanation, the error coping strategy of the hearer plays an essential role. In order to account for implicature cancellation, clausal implicatures, and implicatures of complex sentences with disjunction, an additional nonmonotonic component is needed which represents normality assumptions about the level of speaker expertise.

Keywords: scalar implicatures in complex sentences, error models, game theory, preferential models.

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1. Introduction

> "[I]n all sciences having to do with systems it is a well–known fact that if one wants to get insight into how a system works, it is more revealing to regard instances of small misfunctions than examples of perfect functioning." (Zaefferer, 1977, p. 329)

That error coping strategies should play a role in communication, and especially in pragmatics, is such an obvious idea that it makes one wonder why it has not been explored seriously before. In this paper, we present a model of scalar implicatures that is based on the assumption that hearers posses error coping strategies for speakers who omit parts of their utterances. If for example $\varphi_{\exists\neg\forall}$ says that some but not all of the boys came to the party, the speaker can literally express this message by 'some but not all came' ($F_{\exists\neg\forall}$). If we assume that rising complexity leads to rising costs, then more complex signals will not be considered by the speaker. Overall there are three sentences which a speaker who knows the exact state of affairs can possibly utter: $\{F_{\forall}, F_{\exists\neg\forall}, F_{\neg\exists}\}$. Hence, a speaker who literally expresses his messages can never produce the sentences $F_{\exists}$ and $F_{\neg\forall}$. But if we assume that the speaker may be *lazy* and omit some part of the conjuncts, then $F_{\exists}$ and $F_{\neg\forall}$ can occur in situations in which the speaker intends to communicate the message $\varphi_{\exists\neg\forall}$. We will call $\mathcal{N}_{\varphi_{\exists\neg\forall}} = \{F_{\exists\neg\forall}, F_{\exists}, F_{\neg\forall}\}$ the *noise set* of message $\varphi_{\exists\neg\forall}$. As the other messages are not conjuncts, their noise sets only contain their literal expression, i.e. $\mathcal{N}_{\varphi_{\forall}} = \{F_{\forall}\}$ and $\mathcal{N}_{\varphi_{\neg\exists}} = \{F_{\neg\exists}\}$. The hearer, when receiving signal $F_{\exists}$, can then infer with certainty that the speaker's message was $\varphi_{\exists\neg\forall}$. This is the basic idea which we will spell out in more detail in this paper.

As this idea is novel,[1] special considerations have to be given to its foundations. We outline a game theoretic model which allows us to make explicit the strategic reasoning which establishes the hearer's error coping strategy as a rational best response to the presence of a certain type of lazy speakers.

[1]The framework was first introduced in (Benz, 2012), but only the more elementary examples of implicatures had been treated. The paper also contains applications to relevance implicatures and elementary cases of presupposition accommodation.

Almost all game theoretic explanations of scalar implicatures are based on the assumption that addressees when facing ambiguous signals choose the interpretation which the speaker most probably wanted to express. In addition they have either to assume that the speaker's choice of signals is restricted to a linguistically given set of alternatives, e.g. (Franke, 2009), or that the choice of unambiguous signals like 'some but not all' comes with exceedingly high costs, e.g. (Jäger, 2007). In this paper, we present an approach which dispenses with these assumptions. More precisely, we assume (1) that costs of signals are nominal, i.e. they are so small that they are negligible in comparisons to costs or benefits gained by propositional content; (2) that addressees always have the possibility to react with a clarification request which comes with nominal costs and induces the speaker to provide a maximally informative answer; and (3) that their strategies are robust against certain expected *noisy* speaker strategies. Efficient clarification requests have the effect that addressees do not gamble when interpreting sentences, and therefore remove most but not all ambiguities.

In principle, errors may originate from multiple sources. However, we will restrict considerations to errors committed by the speaker in signal selection. We call the models which describe these errors *error models*. Speaker errors may be expected by both speaker and hearer such that this expectation is part of the common ground. Error models are an attempt to model the effects of such commonly expected errors in pragmatics. Basically, they provide a noise set for each of the possible intended messages, where a noise set is a set of utterances which the speaker may produce under the influence of errors. We show how these models can account for scalar implicatures of complex sentences, and how they can be extended to account for implicature cancellation.

Implicatures of complex sentences have been intensively discussed over the last years, in particular in connection with the debate between *globalism* and *localism* (Chierchia, 2004; Chierchia et al., 2008; Geurts, 2009). They became a rich source for non–trivial examples of scalar implicatures, and thereby provide a standard against which to test and compare different approaches. The following examples are taken from (Sauerland, 2004):

(1) **a)** Kai had the broccoli or some of the peas last night.
$\rightsquigarrow$ Kai didn't have the broccoli and some of the peas last night.
$\rightsquigarrow$ Kai didn't have all of the peas last night.

b) Some of the children found some of their parents.
$\rightsquigarrow$ Some but not all of the children found some but not all of their parents.

If we naively apply the standard theory of scalar implicatures (Levinson, 1983) to (1a), the following problem arises: *some* is part of a scale $\langle$*all, some*$\rangle$, hence the utterance of (1a) should implicate that 'Kai had the broccoli or all of the peas last night' is false; however, this not only implies that Kai didn't have all of the peas but also that he had no broccoli. Hence, the predicted implicature is too strong, and the question arises how the standard theory must be modified in order to account for implicatures of scalar expressions which appear in complex sentences.

Disjunctions in complex sentences pose special problems. It is well known that the implicature from *some* to *not all* presupposes that the speaker is an expert, i.e. that he knows the exact state of the world. If the expert assumption is cancelled, also the implicature is cancelled. Disjunctions of the form $\varphi \vee \psi$ implicate that the speaker neither knows whether φ is true, nor whether ψ is true. Hence, he cannot be a domain expert. An additional mechanism is needed which allows for the cancellation of the expert assumption when encountering a disjunction, and to keep it when encountering e.g. a conjunction. This will be done by adding a nonmonotonic component for implicature suspension. We assume that speakers are experts *normally*. A disjunction will lead to a cancellation of the expert assumption and change the relevant set of expected signals.

In Section 2., we briefly review the meanwhile classical approach of (Sauerland, 2004) which employs generalised scales, and a mechanism for the incremental strengthening of implicatures. In Section 3., we introduce our communication models with feedback and noise. In Section 4., we introduce interpretation games, and show how the hearer's error coping strategy can be calculated. We also show that this hearer strategy always arrives at the implicatures of the speaker's utterances. In this sense, it represents Gricean communication. Finally, in Section 5., we show how our model explains scalar implicatures of complex sentences. We first consider sentences without disjunctions, and then add a nonmonotonic component for explaining implicatures of complex sentences with disjunctions. Section 6. then concludes the discussion.

2. Complex sentences: Sauerland's approach

We are interested in implicatures arising from complex sentences as shown in (1). As background to our proposal, we discuss Sauerland's approach (2004). Classical examples of simple sentences giving rise to scalar implicatures are:

(2) **a)** Some of the boys came to the party.
$\rightsquigarrow$Not all of the boys came to the party.

b) John or Peter came to the party.
$\rightsquigarrow$it is not the case that John and Peter came to the party.

The standard theory of scalar implicatures is based on the assumption that the speaker made a choice between a list of expressions belonging to a *scale*, i.e. a list of expressions ordered according to logical strength (Horn, 1972; Levinson, 1983). The scales relevant for the examples in (2) are ⟨*all, some*⟩ and ⟨*and, or*⟩. In the first example, the speaker had a choice between *some* and *all*. As *all* would have made the sentence more informative, the speaker must have had a reason not to say *all*. If he is an expert and tries to be as informative as possible, then the probable reason is that he believes *all* to make the sentence false. In scalar accounts of implicatures, this reasoning is assumed to be part of the linguistic competence of language users, and the presence of a scalar expression automatically leads to the inference that the corresponding sentences with stronger expressions are false (Levinson, 2000). This provides the correct predictions for (2). If the logical complexity of the sentences rises, the limits of this account become evident, as we have seen in Example (1a).

Two components are crucial to Sauerland's theory: a generalised notion of scalar alternatives, and a consistency based incremental strengthening of implicatures. Sauerland's generalised definition of *scalar alternative* is based on the notion of *scale*. Scales must be given as primitives in our lexicon. They are sequences of expressions $\langle \alpha_0, \ldots, \alpha_n \rangle$ which are ordered according to decreasing logical strength. A sentence ψ is a *one–step alternative* of ϕ if the following two conditions hold: (1) $\phi \neq \psi$; (2) there are scalar expressions α and α' which both occur on the same scale C such that ψ is the result of replacing one occurrence of α in ϕ with α'. A sentence ψ is a *scalar alternative* of ϕ if there is a sequence $\langle \phi_0, \ldots, \phi_n \rangle$ with $n \geq 0$, $\phi_0 = \phi$, and $\phi_n = \psi$ such that for all i, with $1 \leq i \leq n$, ϕ_i is a one–step alternative of ϕ_{i-1}. Implicatures are derived from the notion of scalar alternatives and entailment relations (Sauerland, 2004, p. 374): $\neg\psi'$ is an *implicature* of ψ if the following three conditions hold: (1) ψ' is a scalar alternative of ψ, (2) ψ' entails ψ, (3) ψ does not entail ψ'. We first apply these definitions to the following more simple examples:

(3) **a)** Kai had the broccoli and some of the peas last night. (ψ)

b) It is not the case that Kai ate some of the peas. (ϕ)

In (3a), (ψ') 'Kai had the broccoli and all of the peas last night' is a scalar alternative of ψ which also satisfies the entailment conditions for scalar implicatures. Hence, $\neg\psi'$ is implicated. As ψ is true, it follows with $\neg\psi'$ that 'Kai did not have all of the peas last night.' In (3b), (ϕ') 'It isn't the case that Kai ate all of the peas' is a scalar alternative of ϕ. However, ϕ implies ϕ', hence the entailment condition is violated, and $\neg\phi' \equiv$ 'Kai ate all of the peas' is not implicated.

In order to handle examples with disjunctions, Sauerland has to resort to a technical trick. The problem is that if A and B are parts of conjunctions or disjunctions, then $A \vee B, A, B, A \wedge B$ have to be scalar alternatives; but if A and B are not part of conjunctions or disjunctions, they must not be scalar alternatives. For example, if the sentence is 'Kai had the broccoli' (A), then A should not stand in the scalar alternative relation to 'Kai had the broccoli and the beans,' as this would entail that an utterance of A implicates the falsity of B for arbitrary sentences B. However, if the sentence is (1a), then A should stand in the scalar alternative relation to $A \wedge B$. The trick is to assume that $A \vee B$ is in fact an alternative to the formulae $A \boxed{\text{L}} B$ and $A \boxed{\text{R}} B$ which are defined such that $A \boxed{\text{L}} B$ is true iff A is true, and $A \boxed{\text{R}} B$ is true iff B is true.

We now can turn to Example (1a) 'Kai had the broccoli or some of the peas last night'. $(\varphi_{\exists B})$ 'Kai had some of the peas last night' is a scalar alternative to (1a), and therefore also to $(\varphi_{\forall B})$ 'Kai had all of the peas last night.' As the scalar entailment conditions are satisfied, it follows that (1a) implicates that Kai did not have all of the peas. This provides the correct prediction with respect to $\varphi_{\exists B}$. But what about (A) 'Kai had the broccoli'? A is a scalar alternative to $A \vee \varphi_{\exists B}$ which satisfies the entailment conditions; hence, $\neg A$ should be implicated. This prediction is too strong. To overcome this problem, Sauerland introduces a principle of consistency based incremental strengthening of implicatures. First, if ψ' is a scalar alternative of ψ which satisfies the entailment conditions, then he assumes that the *primary* or *weak* implicature is not $\neg\psi'$ but $\neg K\psi'$ with $K\psi'$ meaning that the speaker knows ψ'.[2] A weak implicature $\neg K\psi'$ is strengthened to the *secondary* or *strong* implicature $K\neg\psi'$ if the strong implicature is consistent with the literal meaning of ψ and all its weak implicatures. For Example (1a), this means that (1a) has the weak implicatures $\neg KA$, $\neg K\varphi_{\exists B}$, $\neg K\varphi_{\forall B}$, $\neg K(A \wedge \varphi_{\exists B})$, and $\neg K(A \wedge \varphi_{\forall B})$. It is easy to see that $\neg KA$ cannot be strengthened to $K\neg A$, as this would entail in conjunction with the semantic content of the utterance $K(A \vee \varphi_{\exists B})$ that $K\varphi_{\exists B}$, in contradiction to the weak implicature $\neg K\varphi_{\exists B}$. However $\neg K\varphi_{\forall B}$ can be strengthened to $K\neg\varphi_{\forall B}$. As knowledge entails truth, it follows that $\neg\varphi_{\forall B}$, i.e. that Kai had not all of the peas.

To account for implicatures of complex sentences with disjunction, it seems to be necessary to include a nonmonotonic component in the model. In Sauerland's model, weak implicatures are strengthened to strong impli-

[2] K is a modal operator. $K\varphi$ says that φ is true in all worlds which are compatible with the speaker's beliefs. Knowledge is understood as true belief. Clearly, $K\neg\varphi \Rightarrow \neg K\varphi$, but not $\neg K\varphi \Rightarrow K\neg\varphi$.

catures automatically if this strengthening is consistent. We can think about the difference between strong and weak implicatures as a difference between expert and non–expert speakers: if $K\varphi$ or $K\neg\varphi$ is true, then the speaker is an expert about φ, and if $\neg K\varphi$ and $\neg K\neg\varphi$ is true, then he is a non–expert about φ. Hence, Sauerland's default strengthening means that the hearer assumes the speaker to be an expert whenever this is consistent with what he already knows. We will replace Sauerland's default strengthening by a system of normality assumptions about the speaker's expertise. This will be implemented in such a way that first the expert assumption is applied, and only if the speaker's utterance is pragmatically or semantically inconsistent with this assumption, the expert assumption is lifted. The details will be presented in Section 5.2.

3. Communication with noise

Although Sauerland's model is simple and quite successful, one would like to see an explanation of implicatures which is embedded in a broader foundational framework. Game theoretic approaches have been developed because they allow explaining pragmatic phenomena from general assumptions about rational agents, but until recently they could not cope with such complex examples as those studied by Sauerland and others. The dissertation by Franke (2009) is the first serious exception to this general claim. We are going to present a novel model in which the hearer's error coping strategy plays an essential role. The general model is based on signalling games. We slightly modify them for our purposes. As we are only interested in signal interpretation, we call the respective games *interpretation games*. Before introducing interpretation games, we want to explain the role of *noise* played in our model. Let us therefore consider the following example:

(4) [Doctor's Appointment] Background: John is known to regularly consult two different doctors, physicians A and B. He consults A more often than B. Then, *S* says: 'John has a doctor's appointment at 4pm. He requests you to pick him up afterwards.' (F_D)

In this situation, the most natural reaction of the addressee H is to ask for the practice where John is waiting for him. In any case, the speaker S, by his utterance, has not communicated to the addressee that John is waiting at A's practice. Yet, this is what most game theoretic models of communication would predict. To see this, let us draw a game tree for the example. We assume that the speaker knows at which practice John is waiting. Let us assume that it is A's practice. In this situation, S had a choice between two utterances: the ambiguous utterance F_D, and an unambiguous utterance F_A which names A's practice explicitly. Likewise in the situation in which he knew that John is waiting at B's practice, he had a choice between saying F_D and F_B which makes B's practice explicit. The addressee's interpretation of the explicit utterances is determined by semantic convention, but for the ambiguous utterance there are two interpretation, namely that John is waiting at A's practice φ_A, and that John is waiting at B's practice φ_B. Communication is successful iff the addressee understands φ_A iff φ_A is the case, and φ_B iff φ_B is the case. This leads to the game tree shown in Figure 1. The payoffs at the end of the tree branches indicate whether communication was successful (1) or unsuccessful (0). The minus sign '$-$' after 1 indicates that the utterances F_A and F_B lead to a slight subtraction of (nominal) costs for their additional complexity.

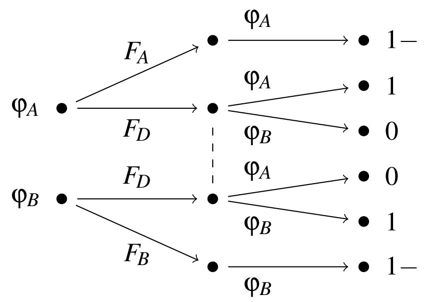

Figure 1: The game tree for the Doctor's Appointment example.

A *strategy* of an agent defines which actions he is choosing in which situation. The choice depends on his *information state*. In the Doctor's Appointment example, the speaker's information state can be identified with either φ_A or φ_B. The hearer's information state can be identified with the signal which he received from the speaker, i.e. with either F_A, F_B, or F_D. Hence, a speaker strategy is a function from $\{\varphi_A, \varphi_B\}$ into $\{F_A, F_B, F_D\}$, and a hearer strategy a function from $\{F_A, F_B, F_D\}$ into $\{\varphi_A, \varphi_B\}$. Such strategies are called *pure* strategies. In general, strategies can be more complicated as speakers and hearers may choose their actions with a certain probability only. In this case, the strategies are called *mixed* strategies. For example, a mixed hearer strategy which interprets F_D half of the time as φ_A, and half of the time as φ_B is represented by $H(\varphi_A|F_D) = 1/2$ and $H(\varphi_B|F_D) = 1/2$. However, for the Doctor's Appointment example it suffices to consider pure strategies. A central notion of game theory is the notion of a *Nash equilibrium*. A strategy pair (S,H) is a Nash equilibrium if (1) the speaker has no interest to switch from S to any other strategy S' if he knows that the hearer sticks to H, and if (2) the hearer has no interest to switch from H to any other strategy H' if he knows that the speaker sticks to S. Furthermore, a Nash equilibrium (S,H) is *strictly Pareto dominating* a strategy pair (S',H') if both speaker and hearer have an interest in switching from (S',H') to (S,H). Finally, a Nash equilibrium is a *strict Pareto–Nash* equilibrium if it is a Nash equilibrium which strictly Pareto dominates all other Nash equilibria.

It is easy to see that the strategy pair (S,H) for which the speaker produces F_D in the more probable situation φ_A, and F_B in the less probable situation φ_B, and for which the hearer interprets F_D as φ_A is strictly Pareto dominating all other strategy pairs. Furthermore, if the hearer takes only the semantic content of the signal F_D into account when choosing his interpretation, he should choose φ_A, as this interpretation is more probable, and therefore carries the higher expected utility. Prashant Parikh (1990, p. 449), assuming that interlocutors agree on Pareto–Nash equilibria, explicitly formulates a principle from which it follows that in situations as that depicted in Figure 1 the ambiguous signal F_D communicates φ_A with *certainty*. Clearly, this is not true for (4). This problem is not confined to Parikh's approach. The same prediction would be reached in similar situations by any other game theoretic framework (Benz and van Rooij, 2007;

Jäger and Ebert, 2009; Franke, 2009). This is simply a consequence of optimising expected utilities which means that an agent who has to decide between two actions with payoffs 0 and 1 will choose the action with certainty which has the higher probability of success.

The natural reaction of the addressee in the Doctor's Appointment example (4) is a clarification request. This possibility is not represented in the game of Figure 1. We therefore extend the game by allowing the hearer to choose an action **c** which leads to a second round of the game. The tree for the extended game is shown in Figure 2. The multiple minus signs signal that nominal costs for producing additional signals must be subtracted. The action **c** represents a clarification request.

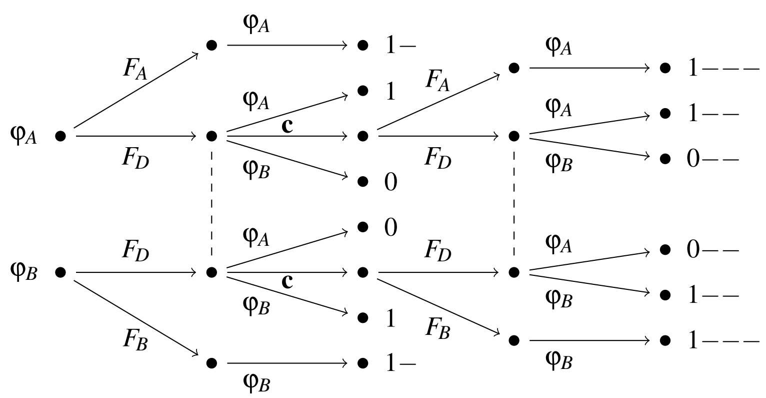

Figure 2: The game tree for the Doctor's Appointment example with clarification request **c**.

We find again that the strategy (S,H) for which the speaker produces F_D in the more probable situation φ_A, and F_B in the less probable situation φ_B, and for which the hearer interprets F_D as φ_A is strictly Pareto dominating all other strategy pairs. Strategies in which the hearer reacts with a clarification request lead to higher costs and are therefore avoided. Hence, simply adding clarification requests to our game does not change the Pareto efficient equilibria, and hence cannot explain why the addressee in the Doctor's Appointment example does not infer from an utterance of F_D that John waits at A's practice. We need another modification of the game.

Obviously, what happened in (4) is that the speaker committed an *error*. This means, he did not pay attention to the fact that he has not mentioned at which practice John is waiting. The addressee infers from the lack of this information that an error has occurred and asks for the missing information. He cannot know whether John is waiting at A's practice and the speaker forgot to mention it, or whether he waits at B's practice. If the costs of communication are negligible in comparison to waiting at the wrong practice, as is surely the case in the Doctor's Appointment example, then responding with a clarification request will be preferred over making a risky choice between φ_A and φ_B. Hence, the possibility of errors, or *noise*, in communication in combination with clarification requests changes the equilibria of the game. It now becomes preferable for the speaker always to send the unambiguous signals F_A and F_B.

When the hearer asks for the practice where John is waiting, then the speaker will become aware of his mistake and avoid it in his answer. We interpret this by the assumption that no errors can occur in the second round, and that therefore speaker and hearer follow the strategy which is Pareto optimal in the original version of the game in Figure 1. This means, that after sending **c** the hearer procures a payoff of 1 minus the additional costs for the clarification request and the answer from the speaker. Hence a clarification request makes sure that the interlocutors receive a payoff of $1 - -$ in situation φ_A, and a payoff of $1 - - -$ in φ_B. In this sense, we call them *efficient*. This also allows to simplify the game tree as shown in Figure 3. For applications, we can ignore the second round, and assume that clarification requests are evaluated immediately.

Signal transmission over noisy channels was intensively studied in electrical engineering and led to the development of information theory (Shannon and Weaver, 1949). Shannon & Weaver studied the effect of noise which corrupts the signal during transmission. We are interested in errors which may be committed by the speaker during signal selection. Our communication model, which is a modification of the Shannon–Weaver communication model (1949, p. 34), is shown in Figure 4.

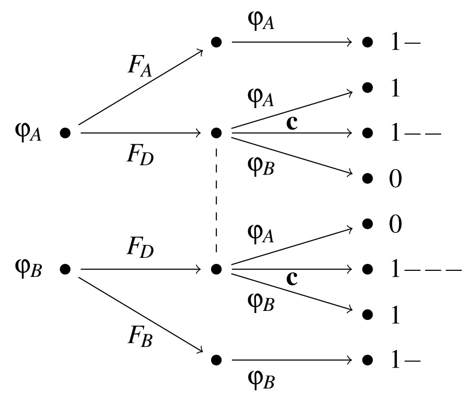

Figure 3: The signalling game for the Doctor's Appointment example with efficient clarification request **c**.

We distinguish two steps in the speaker's production of an utterance. In the first step he selects a message φ which he intends to communicate. In the second step he selects a signal F. This signal F is then reliably transmitted to the hearer. The hearer first has to interpret the signal, which means that he has to choose a message ψ for the signal. Knowing the message, he can then decide about other actions. In this chapter, we are only interested in the interpretation of signals. Hence, we assume that no further actions are chosen, such that interaction stops after signal interpretation. In addition to interpreting the signal, the hearer has the possibility to send feedback in the form of a clarification request. We only consider errors which are committed in signal selection. The noise source and the feedback loop are what distinguishes our model from previous game theoretic models as explored e.g. in (Parikh, 2001; Benz and van Rooij, 2007; Jäger and Ebert, 2009; Franke, 2009).

The game in Figure 2 starts with a given message φ_A. In our communication model of Figure 4 the intended message is chosen by the speaker himself. This choice will depend, among other parameters, on his knowledge about the world, and this in turn on the state of the world itself. Hence, the start node of the tree in Figure 2 has to be divided into three nodes, one for the state of the world, one for the speaker's information state, and one

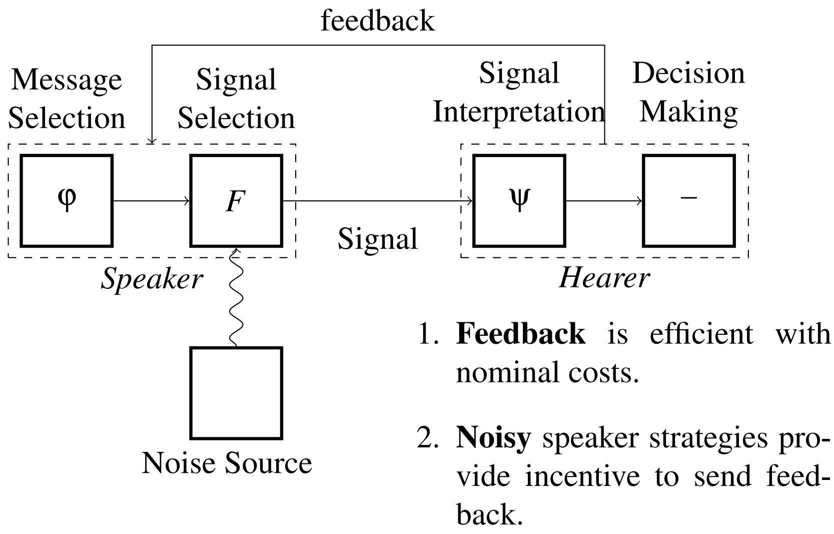

Figure 4: Errors in signal selection

for the chosen intended message. Nature first chooses a world v from a set Ω with probability $P(v)$, then it chooses an information state θ from the set Θ of all possible information states with conditional probability $P(\theta|v)$. Next, the speaker chooses a formula φ from a set of formulae $\mathcal{L}$ with conditional probability $S_{\mathcal{L}}(\varphi|\theta)$, and a signal F from a set of signals $\mathcal{F}$ with conditional probability $S_{\mathcal{F}}(F|\varphi)$. Finally, the hearer chooses a response a to F with conditional probability $H(a|F)$. This response may be the clarification request **c**, or an interpretation $\psi \in \mathcal{L}$. We set $\mathcal{A} = \{\mathbf{c}\} \cup \mathcal{L}$. This leads, if we ignore the possibility of a second round of communication due to clarification requests, to five linearly connected random variables, shown in Figure 5, which together describe the communication process.

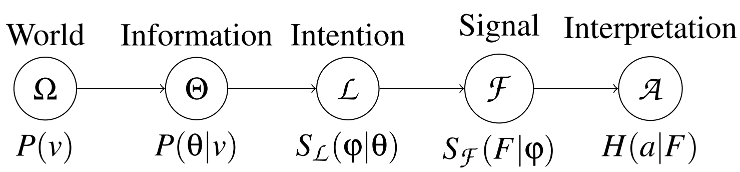

Figure 5: The linear chain of random variables describing communication.

Such a sequence of random variables with conditional probabilities is known as a *Markov chain*. In a game theoretic model, $S_{\mathcal{L}}$ represents the speaker's message selection strategy, $S_{\mathcal{F}}$ his signal selection strategy, and $S(F|\theta) := \sum_{\varphi} S_{\mathcal{L}}(\varphi|\theta)\, S_{\mathcal{F}}(F|\varphi)$ may be called his overall signalling strategy. The hearer's signal interpretation strategy is represented by H. The elements of Θ are called *types* in game theory. We will assume that they are non–empty subsets of Ω as in possible–worlds approaches.

From the conditional probabilities of the Markov chain in Figure 5 a number of useful probability measures of sub–events of the communication process can be derived, as for example the probability of the event of the speaker uttering a specific signal F, or the probability of a certain information state θ. We denote all these probabilities by P as the identity of the probability distributions follows from their arguments. First the probability of a specific course of events $\langle v, \theta, \varphi, F, a\rangle$, which is identical with a branch in the game tree, is defined by:

$$P(v, \theta, \varphi, F, a) = P(v)\, P(\theta|v)\, S_{\mathcal{L}}(\varphi|\theta)\, S_{\mathcal{F}}(F|\varphi)\, H(a|F). \tag{3.1}$$

The probability of a subset of variables taking certain values is calculated by summing up over all other variables. For example:

$$P(\theta) = \sum_{v} P(v) P(\theta|v), \;\; P(v, F) = P(v) \sum_{\theta} P(\theta|v) \sum_{\varphi} S_{\mathcal{L}}(\varphi|\theta) S_{\mathcal{F}}(F|\varphi). \tag{3.2}$$

Finally, conditional probabilities $P(x|y)$ are defined by the quotient of the probability of $P(x, y)$ and $P(y)$. For example:

$$P(v|F) = \frac{P(v, F)}{P(F)}, \;\; P(v, \varphi|\theta, F) = \frac{P(v, \theta, \varphi, F)}{P(\theta, F)}. \tag{3.3}$$

Written in explicit form, this can produce lengthy sums in numerator and denominator. The following conditional probability will be of special importance:

$$P(\varphi|F) = \frac{P(\varphi, F)}{P(F)} = \frac{\sum_v P(v) \sum_\theta P(\theta|v)\, S_{\mathcal{L}}(\varphi|\theta)\, S_{\mathcal{F}}(F|\varphi)}{\sum_v P(v) \sum_\theta P(\theta|v) \sum_\varphi S_{\mathcal{L}}(\varphi|\theta)\, S_{\mathcal{F}}(F|\varphi)}. \tag{3.4}$$

This is the probability of the set of all branches containing φ and F divided by the probability of the set of all branches containing F. This is by definition the probability that a branch contains φ given that it contains F.

The conditional probability $P(x|y)$ tells us how probable it is that x when we have learned that y occurred. We can use this for a very general definition of implicature. We say that the utterance of F *implicates* that some other variable takes on value y, $F \rightsquigarrow y$, iff $P(y|F) = 1$. This means in particular that:

$$F \rightsquigarrow \varphi \Leftrightarrow P(\varphi|F) = 1. \tag{3.5}$$

This is a formal interpretation of Grice' idea that implicatures are what the speaker must have had in mind when making his utterance:

> "... what is implicated is what it is required that one assume a speaker to think in order to preserve the assumption that he is observing the Cooperative Principle (and perhaps some conversational maxims as well), ..." (Grice, 1989, p. 86)

We now know how to describe the communication process, and how to calculate implicatures once the strategies of speaker and hearer are known. What we do not know is how the strategies themselves can be determined.

4. Interpretation games with noise

As we have introduced all parameters of an interpretation game in the last section, we can be very brief about them in this section. We first summarise how an interpretation game is played:

1. Nature chooses a world v and a type θ for the speaker.

2. The speaker chooses a message φ from a set $\mathcal{L}$ of first order sentences.

3. Then he chooses a signal F from a set $\mathcal{F}$ of sentences of natural language. In this step, errors may occur.

4. The signal F is reliably transmitted to the hearer who then has to interpret it. If he chooses a formula $\psi \in \mathcal{L}$ as interpretation, then the game ends. If, instead, he sends a clarification request, the game starts again with signal selection.

5. In the second round, no errors can occur, and the hearer can only choose an interpretation which finally ends the game.

The *interpretation game* can be defined by the set of random variables shown in Figure 5, their conditional probabilities minus the speaker's and the hearer's strategies, plus a *payoff*, or *utility measure*. We add two elements to its definition which we have not yet introduced, a function $[\![\,.\,]\!]$ which defines the meaning of sentences in $\mathcal{L}$, and a function $|\,.\,|$ which translates sentences of natural language into formulae in $\mathcal{L}$:

Definition 1 *A structure* $\langle \Omega, \Theta, P, \mathcal{L}, \mathcal{F}, \mathcal{L}, \mathbf{c}, u, [\![.]\!], |.| \rangle$ *is an* interpreted signalling game with clarification request *if* Ω *is a finite set of possible worlds,* Θ *is a set of non–empty subsets* θ *of* Ω*,* P *a probability distribution over* $\Omega \times \Theta$*,*[3] $\mathcal{L}$ *a set of first order formulae for* Ω*,* $\mathcal{F}$ *a set of natural language sentences,* $\mathbf{c}$ *an element not in* $\mathcal{L}$*,* u *a utility measure which maps the branches of the game tree to real numbers, and* $[\![\,.\,]\!] : \mathcal{L} \longrightarrow \mathcal{P}(\Omega)$ *and* $|\,.\,| : \mathcal{F} \longrightarrow \mathcal{L}$ *are two functions. In addition, the following constraints hold:*

1. $P(v) > 0$ *for all worlds* $v \in \Omega$*, and* $P(v, \theta) > 0$ *implies* $v \in \theta$*.*

2. For all sets $X \subseteq \Omega$ *there exists a* $\varphi \in \mathcal{L}$*:* $[\![\varphi]\!] = X$*, and for all formulae* φ *there is a sentence* F *for which* $|F| = \varphi$*.*

3. With $\delta(\varphi, \psi) = 1 \Leftrightarrow \varphi = \psi$ *and* $\delta(\varphi, \psi) = 0$ *otherwise,* u *can be decomposed as follows for branches without clarification request:*

$$u(v, \theta, \varphi, F, \psi) = \delta(\varphi, \psi) - cost(F). \tag{4.6}$$

[3] As explained before, P can be decomposed into $P(v, \theta) = P(v)\, P(\theta|v)$ with $P(v) = \sum_\theta P(v, \theta)$ and $P(\theta|v) = P(v, \theta)/P(v)$.

With $k(\varphi) = \min\{cost(F) \mid \varphi = |F|\}$, *the utility u can be decomposed as follows for branches with clarification request:*

$$u(v, \theta, \varphi, F, \mathbf{c}, G, \psi) = \delta(\varphi, \psi) - (k(\varphi) + cost(G))\,. \tag{4.7}$$

We assume that costs of signals and clarification requests are nominal *but greater* 0. *For* $k(\varphi)$ *to be always well–defined, we assume that for all non–empty* $X \subseteq \mathcal{F}$ *there exists* $F \in X$ *such that* $0 < cost(F) = \min\{cost(F') \mid F' \in X\}$.

The first condition says that the hearer believes all worlds $v \in \Omega$ to be possible and has true beliefs only. The second condition states that each proposition can be described by a formula, and that each formula can be expressed in natural language. The third condition says that communication is successful if the hearer can arrive that the intended message, and that the overall utility divides into the payoff for successful or unsuccessful communication minus the costs of signalling. In (4.7), it would have been more natural to set $u(v, \theta, \varphi, F, \mathbf{c}, G, \psi) = \delta(\varphi, \psi) - (cost(F) + cost(\mathbf{c}) + cost(G))$. However, this would have allowed for the possibility that sending two ambiguous signals and a clarification request is cheaper than sending an unambiguous signal in the first round. Condition (4.7) excludes this possibility.[4]

Although signals have an unambiguous semantics in this model, they can become ambiguous again due to speaker errors. The assumption that the utility measure u is shared by S and H implicitly represents the Gricean cooperative principle. It means that there is no conflict of interest between speaker and hearer, and that they both consider the same outcomes successful. We have not yet represented the maxims of quantity and quality.

With Lewis (2002) we assume that the meaning of a sentence F is defined by the signalling behaviour of the language community, i.e. F means ψ because the population conventionally interprets F by ψ. In this paper we are not concerned with the question how a convention emerges. What we are interested in is how the conventional strategy changes once errors

[4]The condition is only added to avoid case distinctions which are irrelevant to the examples studied in this paper. In general, violations of (4.7) can occur in communication.

are admitted in signalling selection and hearers are allowed to respond with clarification requests. Hence, we assume that the conventional meaning of a signal F is given by its translation $|F|$. For a given interpretation game this means that a hearer who follows the established convention interprets F by $|F|$, and the speaker who wants to communicate a message ψ will, accordingly, choose a signal F for which $|F| = \psi$. Hence, conventional communication is characterised by the following conditions:

1. *Speaker strategy*: $S_{\mathcal{F}}(F|\varphi) > 0 \Rightarrow |F| = \varphi$.
2. *Hearer strategy*: $H(\psi|F) > 0$ *iff* $|F| = \psi$.

This means that speakers only choose signals which literally express their intended messages.

Given these conditions, it is clear that interpretation games have a trivial solution: the speaker always chooses a signal with minimal complexity which semantically expresses the intended formula:

$$S_{\mathcal{F}}(F|\varphi) > 0 \text{ iff } F \in \{F \,|\, |F| = \varphi \wedge cost(F) = \min\{cost(G) \,|\, \varphi = |G|\}\}. \quad (4.8)$$

Clearly, the strategy pair $(S_{\mathcal{F}}, H)$ is a *Nash equilibrium* given the speaker's intention φ; i.e. if the speaker knows that the hearer follows interpretation strategy H, he has no interest to change to any other strategy than $S_{\mathcal{F}}$, and if the hearer knows that the speaker follows signal selection strategy $S_{\mathcal{F}}$, then he also has no interest to change his strategy. An even stronger result holds: ignoring the costs of signals, $(S_{\mathcal{F}}, H)$ is weakly Pareto dominating all other strategy pairs; i.e. no other strategy pair $(S'_{\mathcal{F}}, H')$ can provide higher payoff than $(S_{\mathcal{F}}, H)$.[5] This trivial solution for interpretation games seems to offer little to the explanation of implicatures in complex sentences. As we will see in Section 4., the appearance is misleading.

As explained before, we concentrate on the effects of noise occurring in signal selection. The noise may, in principle, depend on the values of all

[5] If we take costs of signalling into account, then more efficient strategy pairs may exists. For example, assume that speaker and hearer follow $(S_{\mathcal{F}}, H)$ except that the speaker utters the vowel a instead of the most frequent sentence F, and the hearer interprets a as $|F|$. This new strategy is more efficient. We cannot exclude such strategy pairs.

variables in Figure 5 which appear prior to signal selection, i.e. noise may depend on v, θ, and φ. We assume here that it only depends on the intended message φ. In addition, it may also be the case that the signal F which would normally be chosen by the speaker has an influence on the type of error which might occur. In fact, this will be the case in all our examples. However, without loss of generality, a representation can be choose for which the noise depends on the intended message φ only. For assume that the probability of a signal F' which may arise from a noisy speaker strategy is given as $\eta(F'|\varphi,F)$, and that the speaker would normally, i.e. if no errors occur, choose signals with probability $S_{\mathcal{F}}(F|\varphi)$, then the resulting noisy speaker strategy chooses F' with probability $\widetilde{S}_{\mathcal{F}}(F'|\varphi) = \sum_F S_{\mathcal{F}}(F|\varphi)\,\eta(F'|\varphi,F)$. This probability depends on φ only.

The representation of noise can be very much simplified. We describe the possible noise by a set of signals $\mathcal{N}_\varphi \subseteq \mathcal{F}$. If $F \in \mathcal{N}_\varphi$, then it is assumed that F will occur with positive probability in situations in which the speaker intends to communicate φ. All other signals have probability 0. We will see that the hearer's strategy H is already determined once we know all the noise sets $\mathcal{N}_\varphi$. In particular, it does not depend on the probability with which the errors occur. The predictions of the following models hold true as long as the speaker's actual signal selection strategy $\widetilde{S}_{\mathcal{F}}$ is such that $\mathcal{N}_\varphi = \{F \mid \widetilde{S}_{\mathcal{F}}(F|\varphi) > 0\}$.

We now introduce error models which are an attempt to model the effects of commonly expected errors on the speaker's and hearer's strategy. An error model is an interpretation game together with a message selection strategy, and a sequence of noise sets $\mathcal{N}_\varphi$:

Definition 2 *A triple* $\langle \mathcal{G}, S_{\mathcal{L}}, (\mathcal{N}_\varphi)_{\varphi\in\mathcal{L}} \rangle$ *is an* error model for interpretation games *if*

1. $\mathcal{G} = \langle \Omega, \Theta, P, \mathcal{L}, \mathcal{F}, \mathcal{L}, \mathbf{c}, u, [\![.]\!], |.| \rangle$ *is an interpretation game.*

2. $S_{\mathcal{L}}$ *is a message selection strategy.*[6]

[6]The explicit representation of the message selection strategy is not strictly necessary. It can be represented indirectly by the condition that $P(\varphi) = \sum_v P(v) \sum_\theta P(\theta|v)\, S_{\mathcal{L}}(\varphi|\theta) = 0 \Rightarrow \mathcal{N}_\varphi = \emptyset$.

3. $(\mathcal{N}_\varphi)_{\varphi\in\mathcal{L}}$ *is a sequence of sets* $\mathcal{N}_\varphi \subseteq \mathcal{F}$.

In the following, we write $\langle \mathcal{G}, S_\mathcal{L}, \mathcal{N}_\varphi \rangle$ instead of $\langle \mathcal{G}, S_\mathcal{L}, (\mathcal{N}_\varphi)_{\varphi\in\mathcal{L}} \rangle$.

How should the hearer react to the presence of noise? As we have seen before, the possibility of sending clarification requests takes effect when errors are a possibility. Clarification requests are *efficient* in the sense that they come with nominal costs and guarantee an answer from the speaker which is free of noise, hence, they guarantee maximal expected payoff minus some nominal value. If there is any doubt about the interpretation of the speaker's utterance, it follows that the hearer should respond with a clarification request. If there is no doubt, then the additional costs of clarification requests will entail that the addressee will avoid them. Likewise, the speaker will choose his signal such that no clarification request will follow. Hence, we have to determine the set of signals for which the interpretation is not in doubt. Given noise sets $\mathcal{N}_\varphi$, the set of interpretations which are not in doubt are given by:

$$\tilde{\mathscr{B}}(F) := \bigcap \{\{\varphi\} \mid F \in \mathcal{N}_\varphi \wedge P(\varphi) > 0\}. \tag{4.9}$$

This is the set of all interpretations φ for which there exists a situation in which the speaker intends to communicate message φ for which F may be selected as a signal, possibly by mistake. $P(\varphi)$ is the probability of φ being a massage; i.e. $P(\varphi) = \sum_v P(v) \sum_\theta P(\theta|v)\, S_\mathcal{L}(\varphi|\theta)$.

$\tilde{\mathscr{B}}(F)$ can contain at most one element. If $\tilde{\mathscr{B}}(F) = \{\varphi\}$, then the hearer can infer from receiving F that the speaker intended to communicate φ. If there are two situations, one in which the speaker intends to communicate a formula φ, and one in which he intends to communicate a different formula ψ, and if for both formulae F can be produced with positive probability, then $\tilde{\mathscr{B}}(F)$ is empty. In the latter case, the hearer cannot be sure how to interpret F. Hence, a clarification request is the best response. This leads to a new hearer strategy $\overline{H}$ defined as follows:

$$\overline{H}(\varphi|F) = 1 \textit{ iff } \tilde{\mathscr{B}}(F) = \{\varphi\}, \text{ and } \overline{H}(\mathbf{c}|F) = 1 \textit{ iff } \tilde{\mathscr{B}}(F) = \emptyset. \tag{4.10}$$

We call the strategy $\overline{H}$ the *canonical* hearer strategy of the given error model. For this strategy, the speaker has the possibility to improve his original strategy S by intentionally choosing signals which otherwise could only be produced under the influence of noise. Let $\mathcal{U}$ be the set of all $F \in \mathcal{F}$ for which $\tilde{\mathscr{B}}(F)$ has exactly one element:

$$\mathcal{U} := \{F \,|\, \tilde{\mathscr{B}}(F) \neq \emptyset\}. \tag{4.11}$$

The set $\mathcal{U}$ contains the signals from which the speaker can choose without fear of clarification requests. Hence, the most efficient signal selection strategy against $\overline{H}$ is the strategy which only chooses minimally complex signals from $\mathcal{U}$. With $\bar{k}(\varphi) := \min\{cost(F) \,|\, F \in \mathcal{N}_\varphi \cap \mathcal{U}\}$, the set of minimally complex signals for message φ is:

$$\mathcal{U}_\varphi := \{F \in \mathcal{N}_\varphi \cap \mathcal{U} \,|\, cost(F) = \bar{k}(\varphi)\}. \tag{4.12}$$

If the $\mathcal{U}_\varphi$ are not empty, an improved speaker strategy $\overline{S}_\mathcal{F}$ can be defined as follows:

$$\overline{S}_\mathcal{F}(F|\varphi) = |\mathcal{U}_\varphi|^{-1} \text{ if } F \in \mathcal{U}_\varphi \text{ and } \overline{S}_\mathcal{F}(F|\varphi) = 0 \text{ otherwise.} \tag{4.13}$$

Clearly, the strategy pair $(\overline{S}_\mathcal{F}, \overline{H})$ is at least a weak Nash equilibrium of $\mathscr{G}$, the hearer strategy strongly dominates all other hearer strategies in the presence of the noise characterised by the error model, and $\overline{S}_\mathcal{F}$ as a strategy against $\overline{H}$ is strongly dominating all strategies $S'_\mathcal{F}$ which assign positive probability to signals which could not be produced by $\overline{S}_\mathcal{F}$.

We have seen, that one way of solving the coordination problem posed by interpretation games is to literally express what one intends to express. The resulting equilibrium we denoted by $(S_\mathcal{F}, H)$. The presence of noise pushes the strategy pair for literal communication out of equilibrium and into a new equilibrium. In this new equilibrium, the hearer's strategy tells us how he will cope with expected errors. In particular, it will explain how he arrives at implicatures as will be shown below.

Figure 6 summarises the dynamics induced by noise. The noise sets $\mathcal{N}_\varphi$ represent the noisy speaker strategies $\widetilde{S}_{\mathcal{F}}$ for which $\mathcal{N}_\varphi = \{F \,|\, \widetilde{S}_{\mathcal{F}}(F|\varphi) > 0\}$. A strategy pair $(\widetilde{S}_{\mathcal{F}}, H)$ with literal interpretation strategy H is not in equilibrium, and the hearer will switch to strategy $\overline{H}$, which is his best response to $\widetilde{S}_{\mathcal{F}}$. The resulting strategy pair $(\widetilde{S}_{\mathcal{F}}, \overline{H})$ is also not in equilibrium, and the speaker will switch to $\overline{S}$, which is the best response to $\overline{H}$.

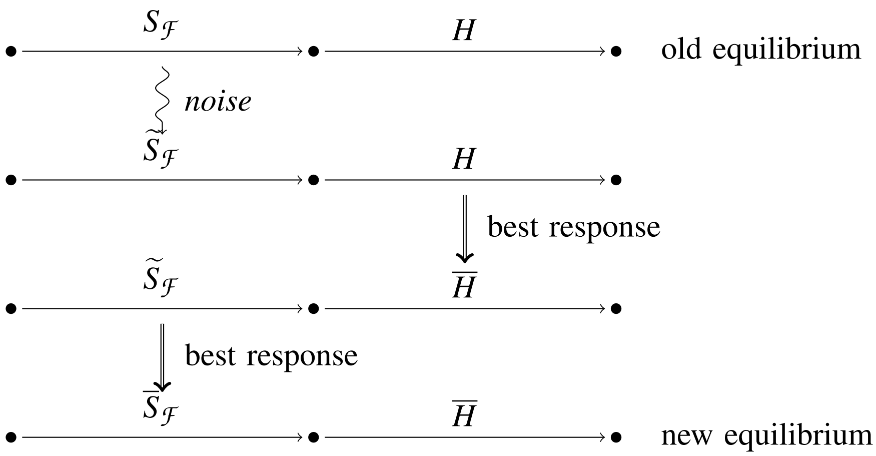

Figure 6: The effect of noise: from old to new equilibrium.

We said that $(S_{\mathcal{F}}, H)$ represents the signalling strategy of literal communication. In a precise sense, $(\overline{S}_{\mathcal{F}}, \overline{H})$ represents the signalling strategy of Gricean communication, i.e. communication with implicatures. This follows from the following theorem:

Theorem 1 *Let $\langle \mathcal{G}, S_L, \mathcal{N}_\varphi \rangle$ be a given error model, and $\overline{H}$ its canonical hearer strategy. Let $\widetilde{S}_{\mathcal{F}}$ be any speaker strategy for which $\widetilde{S}_{\mathcal{F}}(F|\varphi) > 0 \Leftrightarrow F \in \mathcal{N}_\varphi$. Then:*

$$\overline{H}(\varphi|F) > 0 \Leftrightarrow F \rightsquigarrow \varphi. \tag{4.14}$$

Proof: We first show '$\Rightarrow$'. $\overline{H}(\varphi|F) > 0$ is equivalent to $\overline{H}(\varphi|F) = 1$. By definition, it follows that $\varphi \in \tilde{\mathscr{B}}(F)$, hence, that $\exists\theta\, (P(\theta, \varphi) > 0 \wedge F \in \mathcal{N}_\varphi)$, and therefore that $\widetilde{S}_{\mathcal{F}}(F|\varphi) > 0$ and $\widetilde{S}_{\mathcal{F}}(F|\psi) = 0$ for all $\psi \neq \varphi$. This entails

$\sum_{v,\theta} P(v)\, P(\theta|v)\, S_{\mathcal{L}}(\varphi|\theta)\, \widetilde{S}_{\mathcal{F}}(F|\varphi) = \sum_{v,\theta,\varphi} P(v)\, P(\theta|v)\, S_{\mathcal{L}}(\varphi|\theta)\, \widetilde{S}_{\mathcal{F}}(F|\varphi)$. It follows that

$$P(\varphi|F) = \frac{\sum_{v,\theta} P(v)\, P(\theta|v)\, S_{\mathcal{L}}(\varphi|\theta)\, \widetilde{S}_{\mathcal{F}}(F|\varphi)}{\sum_{v,\theta,\varphi} P(v)\, P(\theta|v)\, S_{\mathcal{L}}(\varphi|\theta)\, \widetilde{S}_{\mathcal{F}}(F|\varphi)} = 1.$$

By definition, this entails $F \rightsquigarrow \varphi$.

We next show '$\Leftarrow$'. From $P(\varphi|F) = 1$, it follows that $F \in \mathcal{N}_\varphi$ for some φ with $P(\varphi) > 0$. Suppose there exist a message ψ for which $F \in \mathcal{N}_\psi$, $P(\psi) > 0$, and $\psi \neq \varphi$. Then,

$$\begin{aligned}
&\sum_{v,\theta} P(v)\, P(\theta|v)\, S_{\mathcal{L}}(\varphi|\theta)\, \widetilde{S}_{\mathcal{F}}(F|\varphi) \\
<&\sum_{v,\theta} P(v)\, P(\theta|v)\, \Big(S_{\mathcal{L}}(\varphi|\theta)\, \widetilde{S}_{\mathcal{F}}(F|\varphi) + S_{\mathcal{L}}(\psi|\theta)\, \widetilde{S}_{\mathcal{F}}(F|\psi)\Big) \\
\leq &\sum_{v,\theta,\varphi} P(v)\, P(\theta|v)\, S_{\mathcal{L}}(\varphi|\theta)\, \widetilde{S}_{\mathcal{F}}(F|\varphi).
\end{aligned}$$

Which entails that $P(\varphi|F) < 1$, in contradiction to $F \rightsquigarrow \varphi$. Hence, there is no ψ for which $F \in \mathcal{N}_\psi$, $P(\psi) > 0$, and $\psi \neq \varphi$. Hence, $F \in \mathcal{N}_\psi \wedge P(\psi) > 0$ entails $\varphi = \psi$, and therefore $\tilde{\mathscr{B}}(F) = \{\varphi\}$. From this it finally follows that $\overline{H}(\varphi|F) = 1$. ■

How to apply error models? As we have seen, the hearer's interpretation strategy is determined by the sets $\tilde{\mathscr{B}}(F)$, which in turn depend on the noise sets $\mathcal{N}_\varphi$ and the probabilities $P(\varphi)$ of the messages φ. As $P(\varphi) = \sum_{v,\theta} P(v)\, P(\theta|v)\, S_{\mathcal{L}}(\varphi|\theta)$, the probability $P(\varphi)$ depends on the types which have positive probability, and the messages which can be chosen by the speaker. Hence, a concrete model has to specify the possible information states of the speaker, his message selection strategy, and the noise sets. With these parameters set, the implicatures and interpretations of utterances are completely determined.

5. Implicatures of complex sentences

In this section, we apply error models to describe implicatures of complex sentences. As is well known, disjunctions give rise to so–called *clausal implicatures* (Gazdar, 1979). For example, the utterance of $A \vee B$ gives rise to the implicature that the speaker does neither know whether A nor whether B, which entails that the speaker cannot be a domain expert. Clausal implicatures occur for example in (1a). We will first consider simpler examples in which no clausal implicatures occur, i.e. examples (2) and (3a). For these examples, we can assume that the speaker is an expert. This assumption will be lifted when we consider disjunctions.

5.1 Complex sentences without disjunction

In this section, we consider implicatures of complex sentences without disjunction. In particular, we treat the following examples:

(5) **a)** Kai had some of the broccoli.
$\rightsquigarrow$ Kai had some but not all of the broccoli.

b) Kai had the broccoli and some of the peas last night.
$\rightsquigarrow$ Kai didn't have all of the peas last night.

c) Some of the people had some of their broccoli.
$\rightsquigarrow$ Some but not all of the people had some but not all of their broccoli.

d) All of the people had some of their broccoli.
$\not\rightsquigarrow$ All of the people had some but not all of their broccoli.

The main parameters which we have to determine are the possible speaker's types, his message selection strategy, and the noise sets. In this sub–section, we assume that the speaker is an expert, i.e. he knows the exact state of the world. This is captured by the following condition:

$$\forall v, \theta : P(\theta|v) > 0 \rightarrow \theta = \{v\}. \tag{5.15}$$

This means that we can identify the speaker's type with a world v.

Simplifying matters somewhat, we can say that Grice's maxim of quantity asks the speaker to communicate as much information as possible. This translates into a constraint on the speaker's message selection strategy $S_{\mathcal{L}}$. It says that $S_{\mathcal{L}}$ should select propositions which uniquely characterise the actual world v. However, Grice maxim of quantity is checked by considerations of relevance.[7] As interpretation games only consider signal interpretation without taking into account the further aims of the interlocutors, we cannot make any model internal predictions about how much information is relevant in the given situation. As will become clear below, we cannot do without some representation of the relevant *level of granularity* of information which is required from the speaker. For example, if in (5a) the exact amount of broccoli is at issue, for example if the background question is 'how many grams of the broccoli did Kai eat?', then any answer which only states that Kai ate *some* broccoli will immediately be answered by a clarification request. We encode the appropriate level of granularity by the set $\mathcal{L}$ of formulae from which the speaker can choose his message. For Example (5a) we can assume that the relevant level of granularity is given by the sentences 'Kai had all of the broccoli,' 'Kai had some of the broccoli,' 'Kai had some but not all of the broccoli,' 'Kai had not all of the broccoli,' and 'Kai had none of the broccoli.' The relevant formulae are unambiguously expressed by the respective sentences of natural language. We find the following one–one correspondence between formulae in $\mathcal{L}$ and utterances in $\mathcal{F}$. As before, we use suggestive notation:

$\mathcal{L}$	Meaning	$\mathcal{F}$
$\varphi_{\forall}$	'Kai had all of the broccoli'	$F_{\forall}$
$\varphi_{\exists\neg\forall}$	'Kai had some but not all of the broccoli'	$F_{\exists\neg\forall}$
$\varphi_{\exists}$	'Kai had some of the broccoli'	$F_{\exists}$
$\varphi_{\neg\forall}$	'Kai had not all of the broccoli'	$F_{\neg\forall}$
$\varphi_{\neg\exists}$	'Kai had none of the broccoli'	$F_{\neg\exists}$

[7] We here use the notion of *relevance* in a purely pre–theoretic sense. As we have argued elsewhere, it is not the *relevance* but the expected *utility* of information that is optimised in discourse (Benz, 2006).

The formulae are not equally informative. Hence, we restrict the speaker's choice of messages to the most informative ones which he believes to be true:

$$S_{\mathcal{L}}(\varphi|\{v\}) > 0 \textit{ iff } v \models \varphi \text{ and } \varphi \in \{\varphi_{\forall}, \varphi_{\exists\neg\forall}, \varphi_{\neg\exists}\}. \tag{5.16}$$

This is our equivalent to the Gricean maxims of quality and quantity. It is important to emphasise here that our approach deviates from Neo–Gricean approaches in the tradition of Horn (1972) by the absence of *Horn scales*. That means, we only set a level of granularity for messages and make assumptions about the speaker's preferences over them. The notion of a *scale* does not play any technical role in our approach!

About the costs of signals we only assume that the relations $cost(F) < cost(G)$ reflect the intuitive differences in complexity.

The core of the model are the noise sets $\mathcal{N}_{\varphi}$. As explained in the introduction, we assume here that scalar implicatures are the effect of the speaker's tendency to omit parts of their messages from signalling. From the perspective of literal communication, a speaker who omits parts of the message commits an error. As this error occurs in signal selection, it can be represented by a noise set $\mathcal{N}_{\varphi}$ which contains all signals which result from omitting parts of the literal expression of φ. More specifically, we assume that speakers may omit some *conjunct*. Let $F \lhd G$ hold, iff

1. F results from G by (possibly) omitting some conjunct of G.

2. G is at least as strong as F: $|G| \Rightarrow |F|$.

This means that the following relations hold: $F_{\exists}, F_{\neg\forall}, F_{\exists\neg\forall} \lhd F_{\exists\neg\forall}$. For these simple sentences, the first condition of the definition already implies that if F expresses ψ, and G expresses φ, then $F \lhd G$ implies $\varphi \to \psi$. The second condition will play a role for more complex examples.[8]

[8]This condition may seem unsatisfactory from a cognitive perspective as it asks the speaker to check the logical strength of the messages which are expressed by his utterances. This can be avoided by conditioning the noise sets on the speaker's type θ. But this would introduce additional complexity.

The noise sets will be defined such that they contain all signals which can be produced by a speaker who omits some conjunct of his utterance. To determine this set, we have to know what he would produce if no omissions could occur. As we have argued before, this is determined by the strategy of literal communication. Hence, the noise sets $\mathcal{N}_\varphi$ contain all signals F for which there exists a signal G which literally expresses φ, and for which $F \lhd G$:

$$\mathcal{N}_\varphi = \{F \mid \exists G : \, |G| = \varphi \wedge F \lhd G\}. \tag{5.17}$$

This completes the set up of our first error model. We first consider the basic example (5a) 'Kai had some of the broccoli' ($F_\exists$). The speaker following the literal strategy $S_\mathcal{F}$ can only produce signals $F_{\neg\exists}$, $F_{\exists\neg\forall}$, and $F_\forall$. It is $\mathcal{N}_{\varphi_{\neg\exists}} = \{F_{\neg\exists}\}$, $\mathcal{N}_{\varphi_{\exists\neg\forall}} = \{F_{\exists\neg\forall}, F_\exists, F_{\neg\forall}\}$, and $\mathcal{N}_{\varphi_\forall} = \{F_\forall\}$. As they are disjoint, the set $\mathcal{U}$ of unambiguously interpretable signals is simply the union of these noise sets. The error model then predicts that $\overline{H}(\varphi_{\exists\neg\forall}|F_\exists) = 1$. Hence, a speaker who tries to be efficient can exploit this fact and intentionally produce $F_\exists$ for communicating $\varphi_{\exists\neg\forall}$. This means $\overline{S}_\mathcal{F}(F_\exists|\varphi_{\exists\neg\forall}) > 0$. If we assume that $cost(F_{\neg\forall})$ is higher than $cost(F_\exists)$, it will even hold that $\overline{S}_\mathcal{F}(F_\exists|\varphi_{\exists\neg\forall}) = 1$. As we have seen that $\overline{H}(\varphi_{\exists\neg\forall}|F_\exists) > 0$ implies $F_\exists \rightsquigarrow \varphi_{\exists\neg\forall}$, it follows that 'Kai had some of the broccoli' implicates that Kai had some but not all of the broccoli.

The implicature can also be found using a table as shown in Table 1. The first column lists all possible messages, the second column their literal signals, and the third column their noise sets. A signal is unambiguously interpretable if it occurs in one noise set only. Its interpretation is the message of the row in which the signal occurs. The fourth column shows the signals which unambiguously communicate the respective message.

Next, we consider (5b) 'Kai had the broccoli and some of the peas last night.' For this example, basically the same model can be applied as before. The set of messages seems to be larger at first; i.e. the speaker seems to have a choice between the messages 'Kai had the broccoli and all of the peas last night,' 'Kai had some of the broccoli and all of the peas last night,' 'Kai

ϕ	$Lit(\varphi)$	$\mathcal{N}_\varphi$	$\mathcal{U}_\varphi$
$\varphi_\forall$	$F_\forall$	$F_\forall$	$F_\forall$
$\varphi_{\exists\neg\forall}$	$F_{\exists\neg\forall}$	$F_{\exists\neg\forall}, F_\exists, F_{\neg\forall}$	$F_\exists, F_{\neg\forall}$
$\varphi_{\neg\exists}$	$F_{\neg\exists}$	$F_{\neg\exists}$	$F_{\neg\exists}$

Table 1: *Kai had some of the broccoli.*

had some of the broccoli and some of the peas last night,' etc. But as from the speaker's utterance it logically follows that Kai had the entire broccoli, only the different quantifiers in the second conjunct need to be considered. Hence, if the first conjunct is fixed as (A) 'Kai had the broccoli,' and we write $\exists^!$ for *some but not all* and $\emptyset$ for *none*, then the possible messages are $\varphi_{A\wedge\emptyset B}$, $\varphi_{A\wedge\exists^! B}$, and $\varphi_{A\wedge\forall B}$. This leads to a model which is structurally identical to the model of the basic example (5a). As there, it follows that 'Kai had the broccoli and some of the peas last night' implicates that 'Kai had not all of the peas last night.'

Next, we consider (5c) 'Some of the people had some of their broccoli' ($F_{\exists|\exists}$). As before, we write $\exists^!$ for *some but not all*, and $\emptyset$ for *none*. Assuming the same level of granularity as in the previous examples, we arrive at the following set of messages and signals:

	$\emptyset$	$\exists^!$	$\forall$
$\emptyset$	$\varphi_{\emptyset\|\emptyset}$	$\varphi_{\emptyset\|\exists^!}$	$\varphi_{\emptyset\|\forall}$
$\exists^!$	$\varphi_{\exists^!\|\emptyset}$	$\varphi_{\exists^!\|\exists^!}$	$\varphi_{\exists^!\|\forall}$
$\forall$	$\varphi_{\forall\|\emptyset}$	$\varphi_{\forall\|\exists^!}$	$\varphi_{\forall\|\forall}$

	$\emptyset$	$\exists^!$	$\forall$
$\emptyset$	$F_{\emptyset\|\emptyset}$	$F_{\emptyset\|\exists^!}$	$F_{\emptyset\|\forall}$
$\exists^!$	$F_{\exists^!\|\emptyset}$	$F_{\exists^!\|\exists^!}$	$F_{\exists^!\|\forall}$
$\forall$	$F_{\forall\|\emptyset}$	$F_{\forall\|\exists^!}$	$F_{\forall\|\forall}$

(5.18)

We assume that the quantifier $\exists^!$ is the conjunction 'some but not all', i.e. $\exists\wedge\neg\forall$. Due to the possessive pronoun 'their', the first quantifier always takes scope over the second quantifier. Hence, the translation of the utterances is unambiguous, and we arrive at $|F_{\exists|\exists}| = \varphi_{\exists|\exists}$, $|F_{\exists|\forall}| = \varphi_{\exists|\forall}$, $|F_{\exists^!|\forall}| = \varphi_{\exists^!|\forall}$, etc. As the second quantifier appears in the scope of the first quantifier, we have to adjust the definition of sub–signal $\lhd$ such that it is guaranteed that the respective sub–formula is implied by the original formula. We say that $F_{P'|Q'} \lhd F_{P|Q}$ iff P' is identical to or a conjunct of P and Q' is identical to or a conjunct of Q such that $\varphi_{P|Q} \to \varphi_{P'|Q'}$. It is here where the second condition of $\lhd$ takes effect. This means for example:

$$F_{\emptyset|\forall} \wedge F_{\forall|\exists}, F_{\emptyset|\forall}, F_{\forall|\exists} \lhd F_{\emptyset|\forall} \wedge F_{\forall|\exists}, \quad F_{\exists!|\exists!}, F_{\exists|\exists!}, F_{\exists|\exists} \lhd F_{\exists!|\exists!}. \tag{5.19}$$

But:

$$F_{\exists!|\exists} \not\lhd F_{\exists!|\exists!}, \quad F_{\emptyset|\exists} \not\lhd F_{\emptyset|\exists!}. \tag{5.20}$$

The noise sets remain unchanged, i.e. $\mathcal{N}_\varphi = \{F \mid \exists G\colon\ |G| = \varphi \wedge F \lhd G\}$. As before we also assume that the speaker chooses from the messages which are most informative; i.e. he chooses a message from $\varphi_{\forall|\forall}, \varphi_{\forall|\exists!}, \varphi_{\forall|\emptyset}, \varphi_{\exists!|\exists!}$, etc. The first column of Table 2 shows all possibilities. The second column shows the noise sets, and the third the speaker optimal signal as predicted by the error model. The hearer following strategy $\overline{H}$ will interpret the signals in the last column by the respective formula in the first column.

φ	$\mathcal{N}_\varphi$	$\mathcal{U}_\varphi$
$\varphi_{\forall\|\forall}$	$F_{\forall\|\forall}$	$F_{\forall\|\forall}$
$\varphi_{\forall\|\exists!}$	$F_{\forall\|\exists!}, F_{\forall\|\exists}$	$F_{\forall\|\exists}$
$\varphi_{\forall\|\emptyset}$	$F_{\forall\|\emptyset}$	$F_{\forall\|\emptyset}$
$\varphi_{\exists!\|\forall}$	$F_{\exists!\|\forall}, F_{\exists\|\forall}$	$F_{\exists\|\forall}$
$\varphi_{\emptyset\|\forall}$	$F_{\emptyset\|\forall}$	$F_{\emptyset\|\forall}$
$\varphi_{\exists!\|\exists!}$	$F_{\exists!\|\exists!}, F_{\exists\|\exists!}, F_{\exists\|\exists}$	$F_{\exists\|\exists}$
$\varphi_{\emptyset\|\exists!}$	$F_{\emptyset\|\exists!}$	$F_{\emptyset\|\exists!}$
$\varphi_{\exists!\|\emptyset}$	$F_{\exists!\|\emptyset}, F_{\exists\|\emptyset}$	$F_{\exists\|\emptyset}$
$\varphi_{\emptyset\|\emptyset}$	$F_{\emptyset\|\emptyset}$	$F_{\emptyset\|\emptyset}$

Table 2: *Some of the people had some of their broccoli.*

We can see from the sixth line that $F_{\exists|\exists}$ ‘Some of the people had some of their broccoli’ implicates that some but not all of the people had some but not all of their broccoli ($\varphi_{\exists^!|\exists^!}$). $F_{\exists|\exists^!}$ is not listed in the third column because $\mathcal{U}_\varphi$ collects signals of minimal complexity only; however, the error model predicts that the hearer would interpret $F_{\exists|\exists^!}$ as implying that the speaker intended to communicate $\varphi_{\exists^!|\exists^!}$. These predictions are in accordance with the literature.

Finally, we turn to Ecxample (5d) 'All of the people had some of their broccoli' ($F_{\forall|\exists}$). The third row of Table 2 says that $F_{\forall|\exists}$ implicates that all of the people had some but not all of their broccoli ($\varphi_{\forall|\exists^!}$). This prediction is generally contested, see e.g. (Geurts and Pouscoulous, 2009; Chemla, 2009), and we will see that it vanishes once we drop the simplifying assumption that the speaker's choice is restricted to the messages shown in (5.18). We still assume that the level of granularity is defined by these messages, but the speaker may now also build conjunctions of these formulae in order to characterise the actual world as closely as possible. There are seven possible states of the world. Each world is defined by a conjunction of the negated or unnegated formulae $\varphi_{\exists|\forall}, \varphi_{\exists|\exists^!}, \varphi_{\exists|\emptyset}$:

world	v_1	v_2	v_3	v_4	v_5	v_6	v_7
$\varphi_{\exists\|\forall}$	+	−	−	+	+	−	+
$\varphi_{\exists\|\exists^!}$	−	+	−	+	−	+	+
$\varphi_{\exists\|\emptyset}$	−	−	+	−	+	+	+

(5.21)

For the different worlds there may exist several formulae that describe it. The following Table 3 only shows one formula for each possible world. Rows which are not separated by a horizontal line belong to the same possible world. The alternative messages are represented by their corresponding signals.

The table shows that the speaker never uses the signal $F_{\exists|\exists}$: 'Some of the people had some of their broccoli'.' Hence, it is predicted that the hearer who has to guess the full intended message will react with a clarification request. $F_{\exists|\exists}$ does not tell him whether $F_{\emptyset|\forall}$ 'No one had all of his broccoli' or $F_{\emptyset|\emptyset}$ 'No one had none of his broccoli' are true. This prediction seems to

world	φ	$Lit(\varphi)$
v_1	$\varphi_{\forall\vert\forall}$	$F_{\forall\vert\forall}$
v_2	$\varphi_{\forall\vert\exists!}$	$F_{\forall\vert\exists!}$
		$F_{\forall\vert\exists} \wedge F_{\emptyset\vert\forall}$
v_3	$\varphi_{\forall\vert\emptyset}$	$F_{\forall\vert\emptyset}$
		$F_{\emptyset\vert\exists}$
v_4	$\varphi_{\forall\vert\exists} \wedge \varphi_{\exists!\vert\forall}$	$F_{\forall\vert\exists} \wedge F_{\exists!\vert\forall}$
		$F_{\forall\vert\exists} \wedge F_{\exists!\vert\exists!}$
		$F_{\forall\vert\exists} \wedge F_{\exists\vert\forall} \wedge F_{\exists\vert\exists!}$
		$(F_{\exists!\vert\forall}\wedge)F_{\exists!\vert\exists!} \wedge F_{\emptyset\vert\emptyset}$
v_5	$\varphi_{\exists!\vert\forall} \wedge \varphi_{\emptyset\vert\exists!}(\wedge\varphi_{\exists\vert\emptyset})$	$F_{\exists!\vert\forall} \wedge F_{\emptyset\vert\exists!}$
		$F_{\emptyset\vert\exists} \wedge F_{\exists!\vert\emptyset}$
v_6	$\varphi_{\emptyset\vert\forall} \wedge \varphi_{\exists!\vert\exists!}(\wedge\varphi_{\exists!\vert\emptyset})$	$F_{\emptyset\vert\forall} \wedge F_{\exists!\vert\exists!}$
v_7	$\varphi_{\exists!\vert\forall} \wedge \varphi_{\exists!\vert\exists!} \wedge \varphi_{\exists!\vert\emptyset}$	$F_{\exists!\vert\forall} \wedge F_{\exists!\vert\exists!} \wedge F_{\exists!\vert\emptyset}$
		$F_{\exists!\vert\forall} \wedge F_{\exists!\vert\exists!} \wedge F_{\exists!\vert\exists}$

Table 3: *Some of the people had some of their broccoli.*

be correct. However, in all cases in which $F_{\exists|\exists}$ is an element of the noise sets, it is also the case that $F_{\exists!|\exists!}$ is a sub–formula of the fully intended message. Hence, in this respect, the predictions made on the basis of Table 2 remain valid.

Let us now consider signal $F_{\forall|\exists}$. It is also predicted not to be optimal, in contrast to the situation represented by Table 2. If the speaker produces $F_{\forall|\exists}$, the hearer cannot know whether the speaker intended to communicate $\varphi_{\forall|\exists!}$ or $\varphi_{\exists!|\forall} \wedge \varphi_{\forall|\exists}$. This means, 'Everyone ate some of his broccoli' does not implicate that 'Everyone ate some but not all of his broccoli.' The speaker has to say $F_{\forall|\exists!}$. This prediction seems plausible. Also the other predictions seem plausible, e.g. that $F_{\emptyset|\forall} \wedge F_{\exists|\emptyset}$ 'None of the people ate all of their broccoli and some ate none.' should implicate that some had some of their broccoli. The signals $F_{\forall|\emptyset}$, $F_{\emptyset|\exists!}$, and $F_{\emptyset|\emptyset}$ are problematic because they are difficult to verbalise, and hence are very marked. There are alternative signals which are easier to produce, e.g. 'Everyone either ate all of his broccoli or nothing' ($F_{\forall|\forall\vee\emptyset}$) instead of $F_{\exists^!|\forall} \wedge F_{\emptyset|\exists^!} \wedge F_{\exists^!|\emptyset}$. But we leave formulae as $F_{\forall|\forall\vee\emptyset}$ out of consideration.

5.2 Complex sentences with disjunction

In this section, we discuss complex sentences which contain disjunctions. The main example will be Sauerland's example (1a), repeated here as (6):

(6) Kai had the broccoli or some of the peas last night.

The first problem is that our model predicts that an expert speaker can never produce a disjunction as there is always a stronger alternative namely the conjunction of the two parts of the disjunction. For Example (5b) we assumed that the granularity level is defined by the following three conjunctions: $\varphi_{A\wedge\emptyset B}$, $\varphi_{A\wedge\exists^! B}$, and $\varphi_{A\wedge\forall B}$, where (A) is for 'Kai had the broccoli,', $\exists^!$ for *some but not all*, and $\emptyset$ for *none*. If we add the respective disjunctions, we arrive at the following set of messages:

	$\emptyset$	$\exists^!$	$\forall$
$\wedge$	$\varphi_{A\wedge\emptyset B}$	$\varphi_{A\wedge\exists^! B}$	$\varphi_{A\wedge\emptyset B}$
$\vee$	$\varphi_{A\vee\emptyset B}$	$\varphi_{A\vee\exists^! B}$	$\varphi_{A\vee\emptyset B}$

(5.22)

Clearly, an expert speaker who prefers more informative messages can only choose one of the conjunctions. However, we cannot simply drop the expert assumption (5.15). If we did so, then $\varphi_{\exists}$ would be the most informative message for a speaker who does not know whether $\varphi_{\forall}$ or $\varphi_{\exists\neg\forall}$ is true. We would have to add this type of speaker to Table 1. There would appear a row in which he produced signal $F_{\exists}$, and the implicature $F_{\exists} \rightsquigarrow \varphi_{\exists\neg\forall}$ would be lost.

We have seen that Sauerland's (2004) theory contained two important components: a generalised notion of scalar alternatives, and a consistency based incremental strengthening of implicatures. We replaced the generalised scales by the assumption that the speaker chooses the most informative message of a given granularity level. We have no equivalent to the consistency based incremental strengthening of implicatures. This will be introduced now. We assume that the implicatures are calculated relative to a system of preferential models, such that assertions which are optimal in preferred models are excluded from use in less preferred models.

In logic, preferential models are pairs $\langle \mathcal{M}, < \rangle$ for which $\mathcal{M}$ is a partition of a set of possible worlds, and $<$ a well-founded partial order. For $M, M' \in \mathcal{M}$, $M' < M$ means that M' is preferred over M, or *more normal* than M.[9] A preferential model gives rise to *nonmonotonic* inferences. Informally, an inference from φ to ψ is valid in a preferential model if $\varphi \rightarrow \psi$ is true in all preferred φ–worlds.[10]

Implicatures have often been characterised as a kind of nonmonotonic inferences (Levinson, 2000, Section 1.5). However, we cannot directly use preferential models as we are not interested in preferential inferences which hold between propositions but in inferences from utterance events to propositions which the speaker had in mind when making his utterances. In addition, the specific set up which we choose also has to take into account the different *levels of granularity*. The expert assumption in (5.15) assumes that the speaker is expert about even the most fine–grained distinctions about the domain. In the following, we make a finer distinction within the group of experts and assume that each level of granularity introduces its own class of expert speakers in the sense that they are experts at the respective level of granularity only. Before, we represented a level of granularity by a set of messages $\mathcal{L}$. An expert at the level defined by $\mathcal{L}$ can decide for each message $\varphi \in \mathcal{L}$ whether φ is true or not. In terms of speaker types, this expert assumption is represented by the set of the largest non–empty types which decide all messages in $\mathcal{L}$. It is then a natural generalisation to represent a system of levels of granularity by a sequence $(\mathcal{L}^i)_{i=0,\dots,n}$ of sets of formulae $\mathcal{L}^i$ which are such that being an expert on level j implies being an expert on

[9]Nonmonotonic preferential logics go back to (Shoham, 1987); see also (Kraus et al., 1990; Schlechta, 2004). For the relation to *default logics* see e.g. (Bochman, 2006; Morreau, 1993) who argue that default logics and preferential logics are fundamentally different kinds of nonmonotonic logics.

[10]Let $M(\varphi)$ be the set of the most preferred worlds which satisfy φ, i.e. $M(\varphi) = \bigcup\{M \cap [\![\varphi]\!] \mid M \cap [\![\varphi]\!] \neq \emptyset \wedge \neg\exists M' < M : M' \cap [\![\varphi]\!] \neq \emptyset\}$. Then φ *preferentially implies* ψ iff $M(\varphi) \subseteq [\![\psi]\!]$, i.e. if ψ holds in all preferred φ worlds.

all levels $i \leq j$.

Selecting a level of granularity is part of message selection. Our first normality assumption is that the speaker is supposed to be expert on the level which he has selected. There is no normality assumption about his expert status on the more fine–grained levels. If we assume in line with Grice maxim of quantity that the speaker chooses the most specific message on the respective level of granularity, then this will lead to a partition of each level $\mathcal{L}^i$ into the set of messages $\mathcal{L}^i_e$ which an expert can choose, and the remaining formulae $\mathcal{L}^i_{\bar{e}} = \mathcal{L}^i \setminus \mathcal{L}^i_e$. We assume that these sets are ordered as follows: $\mathcal{L}^i_e < \mathcal{L}^i_{\bar{e}}$, i.e. experts are more normal than non–experts, and $\mathcal{L}^i_{e/\bar{e}} < \mathcal{L}^j_{e/\bar{e}}$ for $i < j$, i.e. the more fine–grained the level of granularity, the less expected are the speakers to be experts. It is in this form of partitions of the set of messages that we use preferential models. The reasons for this specific set up will only become clear with applications.

In our model, we distinguish two levels of granularity. The first level only includes messages which state whether Kai had eaten anything of the different vegetables. On the second level, we include messages which make a statement about the amount of food eaten. The rationale for this distinction is to be found in the different questions which each type of speaker is able to answer. If a speaker is an expert on the first level, then he is able to answer questions of the form 'What did Kai eat?' followed by questions of the form 'Did Kai have broccoli?', 'Did Kai have peas?' For answering these questions, the speaker only needs to have a list with all kinds of vegetable which Kai ate the previous night. On the second level, an expert has to be able to answer questions of the form 'How much of the broccoli did Kai have?', 'How much of the peas did Kai have?' To answer these questions, a list of vegetables which Kai ate the previous night is not sufficient. The speaker, in addition, needs a list which specifies for all vegetables how much Kai ate of them. This difference is only indirectly captured by the sets of messages.

We again use suggestive notation with $\emptyset A$ for 'Kai had no broccoli,' $\exists A$ for 'Kai had some broccoli,' $\emptyset B$ for 'Kai had no peas,' and $\exists B$ for 'Kai had some peas'. As the first level only includes messages which state whether

Kai had eaten anything or not, the formulae with universal quantifier are too informative for this level. Hence, we find the following messages and partitions:

first level	messages	$\mathcal{L}^0$
expert	$\varphi_{\emptyset A \wedge \emptyset B}, \varphi_{\emptyset A \wedge \exists B}, \varphi_{\exists A \wedge \emptyset B}, \varphi_{\exists A \wedge \exists B}$	$\mathcal{L}^0_e$
non–expert	$\varphi_{\emptyset A \vee \emptyset B}, \varphi_{\emptyset A \vee \exists B}, \varphi_{\exists A \vee \emptyset B}, \varphi_{\exists A \vee \exists B}$	$\mathcal{L}^0_{\bar{e}}$

(5.23)

We introduce the second level of messages, which involve judgements about the amount of food eaten, after discussing the noise sets of the first level. We assume that the following additional sub–form relations hold:

1. $\varphi, \psi \lhd \varphi \wedge \psi$,
2. $\varphi \lhd \psi$ then $\chi \vee \varphi \lhd \chi \vee \psi$ and $\varphi \vee \chi \lhd \psi \vee \chi$.

We now turn to Table 4. It only shows the messages which state that Kai had some peas. The second column shows the respective noise sets, and the last column the (minimally complex) elements which are not answered by clarification requests. As we can see, the elements of $\mathcal{U}_\varphi$ are identical to the literal expressions for φ. Hence, no implicatures beyond the literal meaning are generated.

φ	$\mathcal{N}_\varphi$	$\mathcal{U}_\varphi$
$\varphi_{\exists A \wedge \exists B}$	$F_{\exists A \wedge \exists B}, F_{\exists A}, F_{\exists B}$	$F_{\exists A \wedge \exists B}$
$\varphi_{\emptyset A \wedge \exists B}$	$F_{\emptyset A \wedge \exists B}, F_{\emptyset A}, F_{\exists B}$	$F_{\emptyset A \wedge \exists B}$
$\varphi_{\exists A \wedge \emptyset B}$	$F_{\exists A \wedge \emptyset B} F_{\exists A}, F_{\emptyset B}$	$F_{\exists A \wedge \emptyset B}$
$\varphi_{\emptyset A \wedge \emptyset B}$	$F_{\emptyset A \wedge \emptyset B} F_{\emptyset A}, F_{\emptyset B}$	$F_{\emptyset A \wedge \emptyset B}$

Table 4: First level: *Kai had the broccoli or some peas last night.*

The first level seems to be of little interest. This will change when we consider its effect on the speaker's choices on the second level. Here we assume that the speaker is an expert about the *proportion* of each kind of food eaten. We indicate the *part–of* relation which is involved here by adding $\sqsubseteq$ to our notation. For the universal quantifier this will only lead to a more complicated notation. It makes a difference for the existential quantifier $\exists$. The difference between $\varphi_{A\vee\exists B}$ and $\varphi_{A\vee\exists\sqsubseteq B}$ corresponds to the difference between the sentences in (7)

(7) **a)** Kai had the broccoli or some peas last night. ($F_{A\vee\exists B}$)

b) Kai had the broccoli or some of the peas last night. ($F_{A\vee\exists\sqsubseteq B}$)

We will see that our model predicts that only $F_{A\vee\exists\sqsubseteq B}$ implicates $\varphi_{A\vee\exists!\sqsubseteq B}$, and that $F_{A\vee\exists B}$ does *not* implicate $\varphi_{A\vee\exists!B}$.[11] We assume that the signals with '*of*' are more complex synonyms of the signals without '*of*.' We write $F_{\exists\sqsubseteq B}$ for 'Kai ate some *of* the peas", $F_{\exists!\sqsubseteq B}$ for 'Kai ate some but not all *of* the peas', etc. That they are synonyms of the corresponding signals without $\sqsubseteq$ means that they have the same truth conditions. We extend the sub–signal relation such that:

$$F_{\exists\sqsubseteq B}, F_{\exists B} \lhd F_{\exists\sqsubseteq B},\ F_{A\vee\exists\sqsubseteq B}, F_{A\vee\exists B} \lhd F_{A\vee\exists\sqsubseteq B},\ \text{etc.} \tag{5.24}$$

The messages on the second level $\mathcal{L}^1$ are again divided into those which an expert can select $\{\varphi_{A\wedge\forall\sqsubseteq B}, \varphi_{A\wedge\exists!\sqsubseteq B}\}$, and those which he cannot select $\{\varphi_{A\vee\forall\sqsubseteq B}, \varphi_{A\vee\exists!\sqsubseteq B}\}$.

We need to consider one more complicating problem: the two sub–formulae of a message may belong to different levels. For example, the sentence 'Kai had all of the broccoli and some of the peas' expresses the message $\varphi_{\forall\sqsubseteq A\wedge\exists\sqsubseteq B}$, and the sentence 'Kai had all of the broccoli and some peas' expresses the message $\varphi_{\forall\sqsubseteq A\wedge\exists B}$. Hence, the latter message mixes both levels. To handle the mixed cases, we consider the two sub–formulae separately. The first level of granularity about A is defined by $\mathcal{L}^0(A) = \{\varphi_{A\wedge\exists B}, \varphi_{A\wedge\emptyset B}\}$, and the second level by $\mathcal{L}^1(A) = \{\varphi_{A\wedge\forall\sqsubseteq B}, \varphi_{A\wedge\exists\sqsubseteq B}\}$. The mixed cases are then defined by:

$$\begin{aligned} \mathcal{L}^{i\wedge j} &= \{\varphi\wedge\psi \mid \varphi\in\mathcal{L}^i(A)\wedge\psi\in\mathcal{L}^j(B)\} \\ \mathcal{L}^{i\vee j} &= \{\varphi\vee\psi \mid \varphi\in\mathcal{L}^i(A)\wedge\psi\in\mathcal{L}^j(B)\} \end{aligned} \tag{5.25}$$

[11] According to native speaker intuitions, this seems to be correct if 'some' in $F_{A\vee\exists B}$ is unstressed.

We define the preference relation as shown in Table 7. The node $0 \wedge 0$ at the top means that experts about formulae consisting of a conjunction of two primitive formulae of the first level $\mathcal{L}^0$ are more expected than any other type of speaker. The next most expected type of speaker is one who is not an expert about conjuncts of $\mathcal{L}^0$ formulae. This type of speaker is in turn more expected than an expert about conjunctions of mixed formulae. The other nodes have to be read analogously. The marked area is the part of the graph which is relevant to Example 6.

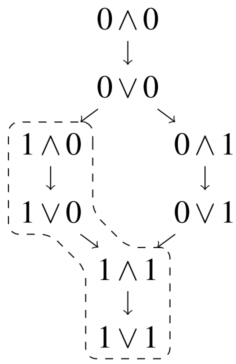

Figure 7: The preferential model.

Finally, we identify the sentence ‘Kai had all of the broccoli and some of the peas’ and ‘Kai had the broccoli and some of the peas’ with each other; i.e. they are both represented by the message $\varphi_{\forall\sqsubseteq A \wedge \exists\sqsubseteq B}$, which we simplify to $\varphi_{A \wedge \exists\sqsubseteq B}$.

The noise sets $\mathcal{N}_\varphi$ are defined as in (5.17). The crucial assumption about the effect of preferential models is the following: When calculating $\tilde{\mathscr{B}}(F)$ for signals F in $\mathcal{N}_\varphi$ only noise sets $\mathcal{N}_\psi$ of messages ψ which are at least as preferred as φ are taken into account. This means for example, if $\varphi \in \mathcal{L}^{1a0}$ and $\psi \in \mathcal{L}^{1a1}$, then $\mathcal{N}_\psi$ is left out of consideration; but if $\varphi \in \mathcal{L}^{1a1}$ and $\psi \in \mathcal{L}^{1a0}$, then $\mathcal{N}_\psi$ is taken into account. An immediate consequence of this assumption is that the sets of unambiguously interpretable signals $\mathcal{U}_\varphi$ on the first level, which are shown in Table 4, do not change when adding higher levels.

Table 5 shows the messages, noise sets, and the sets of uniquely interpretable signals relevant for Example 6. The second column shows to which partition $\mathcal{L}^{1\wedge 0}$, $\mathcal{L}^{1\vee 0}$, $\mathcal{L}^{1\wedge 1}$, or $\mathcal{L}^{1\vee 1}$ the respective message belongs.

	partition	message φ	$\mathcal{N}_\varphi$	$\mathcal{U}_\varphi$
1.	$\mathcal{L}^{1\wedge 0}$	$\varphi_{A\wedge\exists B}$	$F_{A\wedge\exists B}$	$F_{A\wedge\exists B}$
2.		$\varphi_{A\wedge\emptyset B}$	$F_{A\wedge\emptyset B}$	$F_{A\wedge\emptyset B}$
3.	$\mathcal{L}^{1\vee 0}$	$\varphi_{A\vee\exists B}$	$F_{A\vee\exists B}$	$F_{A\vee\exists B}$
4.	$\mathcal{L}^{1\wedge 1}$	$\varphi_{A\wedge\forall\sqsubseteq B}$	$F_{A\wedge\forall\sqsubseteq B}, F_{A\wedge\forall B}$	$F_{A\wedge\forall B}$
5.		$\varphi_{A\wedge\exists!\sqsubseteq B}$	$F_{A\wedge\exists!\sqsubseteq B}, F_{A\wedge\exists\sqsubseteq B}, F_{A\wedge\exists B}$	$F_{A\wedge\exists\sqsubseteq B}$
6.	$\mathcal{L}^{1\vee 1}$	$\varphi_{A\vee\exists!\sqsubseteq B}$	$\varphi_{A\vee\exists!\sqsubseteq B}, F_{A\vee\exists\sqsubseteq B}, F_{A\vee\exists B}$	$F_{A\vee\exists\sqsubseteq B}$

Table 5: *Kai had the broccoli or some peas last night.*

As the first conjunct is fixed (A), we only considered sub–signals which simplify the second conjunct or disjunkt related to the peas (B). Hence, the table only shows the effect of leaving out the preposition 'of' for the part–of relation ($\sqsubseteq$). As on the levels represented by $\mathcal{L}^{1\wedge 0}$ and $\mathcal{L}^{1\vee 0}$ the part–of relation is not represented, it cannot be omitted.

In line four, neither $F_{A\wedge\forall\sqsubseteq B}$ nor $F_{A\wedge\forall B}$ occurs in the noise set of any other message. As the speaker prefers the less complex signal $F_{A\wedge\forall B}$, only this signal belongs to $\mathcal{U}_{\varphi_{A\wedge\forall\sqsubseteq B}}$. In line five, both $F_{A\wedge\exists\sqsubseteq B}$ and $F_{A\wedge\exists B}$ appear in the noise set of $\varphi_{A\wedge\exists!\sqsubseteq B}$. However, $F_{A\wedge\exists B}$ is the optimal signal for $\varphi_{A\wedge\exists B}$ which belongs to the preferred $\mathcal{L}^{1\wedge 0}$. Hence, it is excluded from $\mathcal{U}_{\varphi_{A\wedge\exists!\sqsubseteq B}}$, and only $F_{A\wedge\exists\sqsubseteq B}$ remains. This predicts the following implicatures and non–implicatures:

(8) **a)** Kai had the broccoli and some peas last night. ($F_{A\wedge\exists B}$)
$\not\rightsquigarrow$ Kai didn't have all of the peas.

b) Kai had the broccoli and some of the peas last night. ($F_{A\wedge\exists\sqsubseteq B}$)
$\rightsquigarrow$ Kai didn't have all of the peas.

The sixth line is parallel to the fourthe line. This leads to the predictions:

(9) **a)** Kai had the broccoli or some peas last night. ($F_{A\vee\exists B}$)
$\not\rightsquigarrow$ Kai didn't have all of the peas.

b) Kai had the broccoli or some of the peas last night. ($F_{A\vee\exists\sqsubseteq B}$)
$\rightsquigarrow$ Kai didn't have all of the peas.

We close this section by a short discussion of the message $\varphi_{A\vee\exists\sqsubseteq B}$ which also seems to belong to $\mathcal{L}^{1\vee 1}$. Its literal expression is $F_{A\vee\exists\sqsubseteq B}$. Hence, it seems to make this signal ambiguous. However, if the speaker only knows $\varphi_{A\vee\exists\sqsubseteq B}$, then he only knows $\varphi_{A\vee\exists B}$, and therefore is a non–expert on a lower mixed level ($\mathcal{L}^{1\vee 0}$). He therefore should choose the simpler formula $\varphi_{A\vee\exists B}$. This would lead to the utterance of $F_{A\vee\exists B}$. Hence, the formula $\varphi_{A\vee\exists\sqsubseteq B}$ is not a formula which the speaker might choose. A similar argument tells against the formula $\varphi_{A\wedge\exists\sqsubseteq B}$ as a possible message in situations in which the speaker only knows $\varphi_{A\wedge\exists B}$.

6. Some concluding remarks

If we compare our model with Sauerland's, then, apart from the obvious fact that our model is based on game theory and Sauerland's is not, there are two major differences which we want to highlight here: one concerns the role of linguistic scales, and the other the nature of nonmonotonicity built into the models. To start with the latter, Sauerland assumes an incremental, consistency based strengthening of implicatures from 'the speaker does not know φ' to 'the speaker does know that $\neg\varphi$.' This strengthening happens by *default*. In contrast, our model represents normality assumptions by a variation of *preferential models*. Utterances are interpreted under the assumption that speakers are experts. If the choice of utterance contradicts this assumption, then the expert assumption is cancelled. It has been argued that there is a fundamental differences between these two types of nonmonotonic logics (Morreau, 1993; Bochman, 2006), and, in particular, that preferential models are what is appropriate for modelling normality assumptions (Bochman, 2006). The use of preferential models has the effect that strengthening does not automatically follow from consistency with what is known. However, our presentation of these models had to be complicated by the additional distinction between different levels of fine–grainedness, and the possibility of mixed formulae which contain sub–formulae of different complexity. Hence, further investigation into the nonmonotonic component is necessary. As we indicated, the different levels of granularity seem to be derivable from different sets of questions which can be answered at the different levels.

For the role of scalar alternatives, we want to emphasise that there is no equivalent notion in our model. For example, the speaker can freely choose between sentences 'Some of the boys came to the party,' 'Some boys came to the party,' 'Some but not all of the boys came to the party,' 'Some but not all boys came to the party,' 'All of the boys came to the party,' 'All boys came to the party,' 'None of the boys came to the party,' etc. Costs for these sentences may differ slightly, but the differences are never large enough such that the costs could induce the speaker to choose risky signals. This is probably the most remarkable difference to classical accounts. This also sets it apart from previous game theoretic accounts, e.g. (Jäger, 2007; Franke, 2009).

In the Gricean tradition, scalar implicature are considered generalised implicatures. In the neo-Gricean view, this became to mean that they are nonmonotonic inferences which depend on the logical form of utterances only (Levinson, 1983, 2000). The present approach treats them as contextually triggered inferences. However, this is not directly in conflict with the Neo–Gricean view. As the hearer strategy is modelled as a function from utterances into interpretations, it follows that the implicatures must be inferable from the utterances themselves. In a second step, the interlocutors can learn which sentence patterns trigger which scalar implicatures, and, hence, over time, the inference can become automatic.

In general, there is much more social interaction in communication than can be captured by the simple and abstract models of signalling games. The extension of the present approach to more intricate problems of speech act theory needs, however, a more elaborate representation of social and cognitive parameters, as the insightful discussions in this volume (Gärtner, Evans) show.

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